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GUIDANCE OF BALLISTIC MISSILES
(Chapters 4-5)

by

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GUIDANCE OF BALLISTIC MISSILES (Chapters 4-5)

By: A. M. Zhakov and F. A. Pigulevskiy

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ABSTRACT: The book is intended for officers of the Soviet Armed forces in the fields of engineering and technology and for rocketry students. The authors discuss ballistic-missile guidance in detail. The first chapter covers ballistic-missile trajectories, target accuracy, and rocket dispersion, shown in tables and diagrams, as well as the lateral motion and ranges of ballistic missiles. In the second and third chapters the authors discuss guidance theory, angle of stabilization, and the control of motion dynamics. The last four chapters deal with electronic systems, velocity and position measurements, and command transmissions from control centers. The book has numerous diagrams and illustrations.
English translation: 60 pages.

TABLE OF CONTENTS

* 45 - Actual pages translated: 1--4 and 146--210.

TABLE OF CONTENTS

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

ANNOTATION

Discussed in the book is basic information on the guidance of ballistic missiles. The trajectory of rockets, dispersion of points of their fall, the angular stabilization of rockets, and principles of operation of electronic and autonomous control systems are examined. There is given information about the most important elements of the control equipment. Much space in the book is given to electronic systems, and autonomous control systems are described in less detail.

The book is intended for engineering-technical officiers, officers of the reserve and other persons studying rocket technology.

PREFACE

The development of rocket technology is associated with names of many outstanding Russian and Soviet scientists — K. E. Tsiolkovskiy, K. I. Konstantinov, I. V. Meshcherskiy, F. A. Tsander and others; these scientists created the basis of the theory of jet propulsion and took the first practical steps in rocket building.

The attention of the Communist Party and Soviet government toward the development of domestic science and for strengthening of the Armed Forces promotes considerable progress in the rocket technology in our country.

Triumphs of our achievements in rocket building were the historical launching of the first (in the world) artificial earth satellite, the space station photographing the opposite side of the Moon, the flight of ships with astronauts on board and other later achievements in the mastering of outer space, which provided the Soviet Union with the leading place in the development of rocket technology.

The increasing importance of rockets created the necessity of the publication of books intended for engineering-technical officers, officers of the reserve and other persons dealing with rocketry. In these books principles of operation and the equipment of different systems entering into the complex of the rocket weapon should be examined. This pertains, in particular, to principles of construction and operation of guidance systems of rockets. During the last few years different publishing houses produced several books of this kind, but there still are no books in which problems of guidance would be discussed in application to the most important form of rocket weapon — ballistic missiles of medium and great range. These questions are dealt with in the present book.

Chapter 1 examines trajectories of ballistic missiles and the accuracy of hitting the target (dispersion of rockets). Graphs are given allowing the calculation of elements of the trajectory of the rocket and error of the hit. In this chapter methods are examined of guidance by lateral motion and flying range of ballistic missiles.

Chapters 2 and 3 deal with principles of stabilization of the angular position of the rocket in space and dynamics of control of its motion in a

powered-flight trajectory. For readers not familiar with automation in Chapter 2 information is given on the theory of control which is necessary for understanding of the given material.

The subsequent four chapters are devoted to electronic and autonomous guidance systems of ballistic rockets and to the most important elements of these systems. Different methods of the measurement of coordinates and speed of the rocket and also methods of transmission to the rockets of command guidance are examined. In conclusion there is given a brief comparative characteristic of autonomous (inertial) and electronic control systems.

The book should help in the study of principles of operation and equipment of guidance systems of ballistic missiles. The authors tried to make the book accessible not only for engineers, but also for readers with middle technical education. Therefore, the authors avoided using complicated mathematical derivations and wherever possible gave simplified graphic explanations of obtained dependences. The description of the ballistic missile guidance equipment is based on data from technical literature, the list of which is given at the end of the book.

Chapters 1, 3, 4, 7 and the Introduction were written by A. M. Zhakov, and Chapters 2, 5, 6 and Appendex 1 by F. A. Pigulevskiy.

CHAPTER 4

CONTROL OF MOTION OF THE CENTER OF MASSES OF THE ROCKET. SYSTEMS OF LATERAL RADIO CORRECTION

§ 4.1. Autonomous Control and Remote Control

One of the problems of flight control of a ballistic missile on the powered-flight trajectory is to continuously hold the rocket in the guidance plane, i.e., in a vertical plane passing through the calculation point of the turning off of the einpines. This problem is impossible to solve only with the help of an automatic machine of angular stabilization. Actually, gyroscopic instruments, forming the basis of the automatic machine of stabilization, permit revealing the change in angular position of axes of the rocket. If, however, the rocket deviates from the guidance plane so that its longitudinal axis remains parallel to this plane (Fig. 4.1), then the gyroscopic instruments will not reveal the error z. Furthermore, the equipment of stabilization of the axes cannot

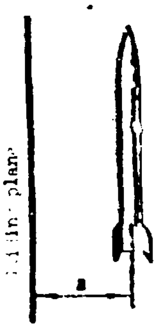

Fig. 4.1. Lateral deviation z of the rocket from the guiding plane.

operate absolutely accurately. There is always an error, let us assume small, in the position of the longitudinal axis of the rocket, i.e., a deviation along course ψ. Due to such deviation the rocket in the course of time even more departs from plane of guidance, i.e., the lateral error z (if the sign of the error along the course does not change) will be increased with the advance of rocket to the point of the turning off of the engine. In this

there appear properties of the rocket mentioned in § 3.1. The automatic machine of angular stabilization changed the character of only the rotation of the rocket around the center of masses. In the lateral motion of the center of masses of the rocket as before there appear properties of the integrating unit. Consequently, gyroscopic equipment for stabilizing axes of the rocket should be augmented by equipment of drift correction of the rocket from the plane of guidance. For this it is necessary to measure coordinates, of the center of masses of the rocket and control the change of these coordinates. Let us investigate the methods of solution of such a problem. Here it will be useful to examine the problem more deeply, not connecting it with the guidance of only ballistic missiles. Therefore, we will consider that target position datum can be changed, i.e., let us take, for example, an air target.

All control systems of rockets can be divided into two classes — system of autonomous control and a system of remote control. In systems of remote control there are used signals proceeding to rocket from the target or from the control center. In autonomous systems control of the rocket is conducted without the entering of such signals.

Belonging to autonomous control systems can be inertial systems, which are built in the following way. On board the rocket there is measured acceleration undergone by the rocket during its movement along the trajectory. Signals proceeding from the accelerometer are integrated. As is known, the speed is equal to the integral from acceleration with time and the path, the integral from speed. Therefore, after the first integration of signals produced by the transducer of acceleration, we obtain the speed of the rocket. Repeated integration of these signals gives the path passed by the rocket. Thus the coordinates of the rocket in space are determined. Moving values of coordinates of the rocket are compared with their program values embodied in the memory of the rocket computer. After exposure of deviations of the rocket from the assigned trajectory there are formed controlling signals, which proceed to the controls.

Technical details of the construction of inertial systems for us now are not of importance. Important only is the fact that the equipment of these systems, located on board the rocket, controls the flight without the entering of some signals from the target or from the control center.* Therefore, inertial systems belong to the class of autonomous systems.

If for operation of inertial systems (as other autonomous systems) there are not required channels of the transmission of information between the rocket and the target or between the rocket and control center, then in systems of remote control making up a second class of control systems of rockets such channels certainly exist. In general (with a mobile target) the equipment of the system of remote control solves four problems:

1) measures moving coordinates and speed of the rocket;

2) measures moving coordinates and speed of the target;

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*One can assume that in autonomous systems the control center functions only prior to the launch of the rocket: instruments of the control center serve for data input about the flight program into the flight equipment of the rocket.

3) compares measured values and produces command signals intended for control of the rocket;

4) transmits command signals to rockets.

For a solution of the first, second and fourth problems there are required channels of the transmission of information (communication channels). The process of the measurement of target position data is essentially the process of transmission of information on the position of a target to the point in which there is the measuring equipment (in the control center of the rocket). In exactly the same way the measurement of coordinates of the rocket is accompanied by the transmission of information on the position of the rocket to the control center. Consequently, in the system of remote control there should be two communication channels along which to the control center proceeds information on coordinates of the rocket and target: the channel of the measurement of coordinates of the rocket and channel of the measurement of target position data. To solve the fourth problem one more communication channel is necessary — the channel of transmission to rocket of commands of control (Fig. 4.2).

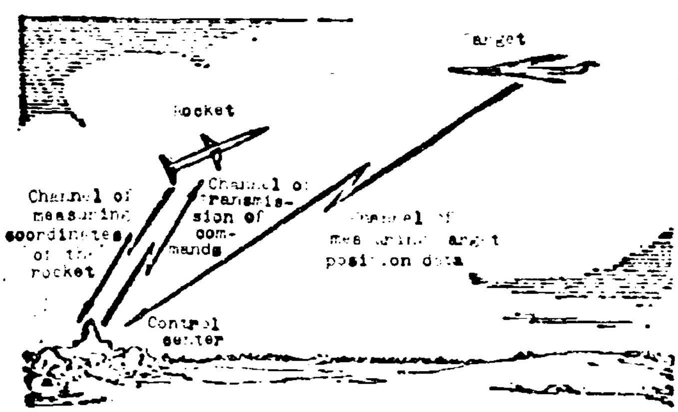

Fig. 4.2. Channels of transmission of information in the remote control system (guidance system).

Remote control systems can be divided into systems of guidance and homing guidance. In guidance systems, the principle of construction of which was just now described (Fig. 4.2), command signals proceed to the rockets from the control center. Systems of homing guidance differ in that in them target position data are measured by flight equipment of the rocket and command signals are produced also on the rocket. In these systems there exists only one communication channel, the channel of the measurement of target position data (Fig. 4.3).

Common for all systems of remote control, including systems of guidance and homing guidance, is the fact that in the control of the rocket there take part communication channels connecting the rocket either with the target or with the control center. In radio control systems which belong to remote control systems, these channels are created with the help of electronic means. For example, the coordinates of the rocket and target can be measured by radar, and the commands to rockets can be transmitted by command radio link.

Ballistic missiles on the powered-flight trajectory can be controlled

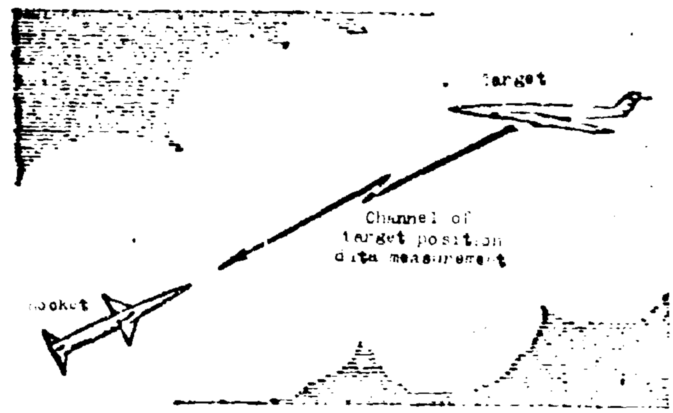

Fig. 4.3. Channel of transmission of information in the homing guidance system.

by electronic guiding systems but built from a simpler diagram than that depicted in Fig. 4.2. Simplification is caused by the fact that with the control of ballistic missiles the target is stationary, and its coordinates are not measured but are introduced by calculations. Therefore, the channel of the measurement of target position data in guidance systems of ballistic missiles is absent.

Let us clarify what distinctions in properties of systems of autonomous control and systems of remote control (in particular, systems of radio control) appear because of the fact that in some of them there are and in others there are not channels of information transmission. First of all, in autonomous systems it is impossible to change the program of flight of rocket after launch. The trajectory of the rocket with autonomous control is calculated and assigned beforehand, and the flight equipment measures the deviation of the rocket from the program trajectory and eliminates these deviations. Consequently, such systems are useful for the guidance of rockets intended for destruction of only fixed targets.

In systems of radio control the trajectory can be definitized during the flight of the rocket to the target with the help of signals proceeding to the rocket by channels of information transmission. Consequently, the system of radio control can be used to destroy not only fixed but also mobile targets. A change in the direction or speed of the target after launching of rocket can be considered by the system which will change the trajectory of the rocket in such a manner that the target is destroyed.

Ballistic missiles are intended for strikes at fixed targets. Therefore, as a rule, it is doubtful whether a change in the assigned trajectory of these rockets after launching will be necessary. However, the communication channel between the rocket and control center can be useful here, since it makes the flight equipment of th[illegible]ket simpler and permits increasing the accuracy of control (see below § 7.4).

The use of communication channels in radio control systems leads also to definite deficiences of these systems as compared to systems of autonomous control. The operation of communication channels is accompanied by the transmission and reception of radio signals, and this lowers the secrecy and noiseproof feature of the control. The enemy, at least in principle, can investigate

the parameters of the transmitted signals, create disturbing radiation and disrupt the course of control of the rocket. Autonomous systems (for example, inertial) are not subject to the action of external interferences.

For flight control of ballistic missiles in powered-flight trajectory radio control systems, of two types are used: guidance systems, by equisignal zone, usually called systems of lateral radio-correction [2, 4] and radio control command systems. Let us investigate the principle of action of these systems.

In systems of lateral radio-correction control of the rocket is fulfilled with the help of the equisignal zone. To obtain the equisignal zone there are used directional transmitting antennas, which possess the property to radiate radio waves not evenly in all sides but chiefly in any one or several directions. Figure 4.4 shows the approximate antenna radiation pattern of

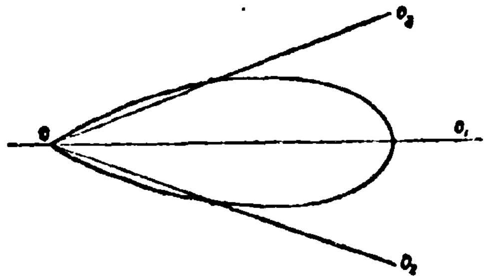

Fig. 4.4. Antenna radiation pattern of the antenna.

the transmitting antenna utilized in the system of lateral radio-correction. The figure shows that in direction OO_1 the antenna radiates the greatest power, and in directions OO_2 there is radiated twice lesser power, etc. For the width of the antenna radiation pattern of antenna it is possible to take the angle between the two rays OO_2, corresponding to the radiated power, equal to half of the maximum value. For high-directional antennas the width of the antenna radiation pattern can not exceed units and even fractions of a degree.

In order to obtain an equisignal zone, one must have an antenna system of a radio transmitter with two antenna radiation patterns 1 and 2, overlaying one another (Fig. 4.5). Along line OO_1, passing through the point of crossing of the diagrams the radiated power is identical for both antennas. Therefore, if the point of reception of the signals is on line OO_1, then the level of the signals at the output of the receiver will also be identical. Direction OO_1 on Fig. 4.5 can be called the equisignal line. In reality the measuring circuits of levels of radio signals possess a threshold of sensitivity and cannot reveal small distinctions in he two accepted signals. Therefore, the equisignal zone inside which signals of antennas 1 and 2 during reception have an identical value constitutes the narrow sector shaded on Fig. 4.5. Line OO_1 is disposed in the middle of this sector (equisignal zone).

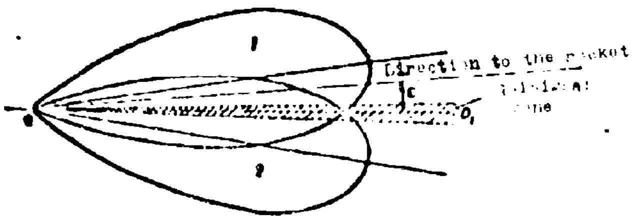

Fig. 4.5. Formation of the equisignal zone.

Radio reception of the device catching the signals of antennas 1 and 2 can be established on the rocket and the middle of the equisignal zone combined with guiding plane. Then the equality of levels of signals will indicate that the drift of the rocket from the guiding plane is absent (or is small).

If the rocket deviates from the equisignal zone (from line OO_1 on Fig. 4.5) to angle ε, the equality of the signals will be disturbed. With deviation to the left signals of antenna radiation pattern 1 will predominate over signals of diagram 2; with deviation to the right there will be observed an opposite relationship. The larger the angle ε, the greater the difference in levels of the signals. Thus if one were to take signals of the ground transmitter on the rocket, then by the signals it is possible to determine whether the rocket is in the direction of the equisignal zone or is deflected aside. In the last case it is possible to determine the amount of deviation and also its side (to the right or to the left).

The system of lateral radio-correction includes the ground transmitting station and flight equipment of the rocket (Fig. 4.6). The ground station creates the equisignal zone combined with the guiding plane. In the flight equipment of rocket there is the receiving-measuring device, which takes signals of the ground transmitter, determines by these signals the value and side of deviation of the rocket from the guiding plane and produces a voltage which then proceeds to the automatic machine of angular stabilization of the rocket. Systems of lateral radio-correction are the subject of subsequent paragraphs of this chapter.

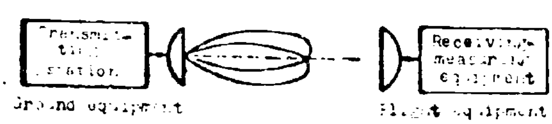

Fig. 4.6. System of lateral radio-correction.

The equipment of the command radio control system consists of a measuring complex, computer and command radio link (Fig. 4.7). The measuring complex can include one or several ground stations, which determine the coordinates and speed of the rocket. In the measurement of these parameters there is usually radio responder included in the flight equipment [39].

The computer, as a rule, is an electronic digital computer, which compares the measured parameters of the motion of rocket with their program values, and in case of deviations exceeding set limits, it produces a command. Here there can be formed commands of the control of lateral motion of the rocket

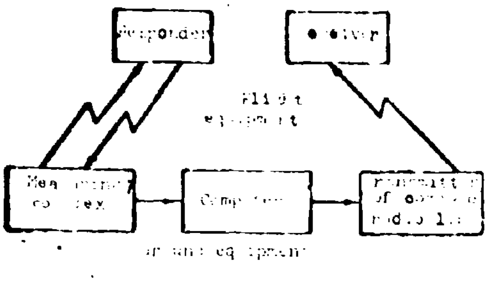

Fig. 4.7. Command system of radio control.

and commands for turning off the propulsion system (range control). Control signals are transmitted to the rocket by the command radio link. Transmitting equipment of the command radio link is at the control center, and the receiving device is located on the rocket.

Methods of the measurement of coordinates and speed of the rocket in radio control command systems are described in Chapter 5. Questions of the transmission of commands are discussed in Chapter 6. Let us note that electronic channels intended for measurement of parameters of the motion of the rocket and transmission of commands can be united into one common (combined) radio channel [13, 32].

§ 4.2. Systems of Lateral Radio-Correction

The equisignal zone, designating in space the guiding plane, in which the rocket should be held in a powered-flight trajectory, is obtained with the help of two covered lobes of the antenna radiation pattern of the transmitting antenna. In the actual equipment for the creation of overlapping antenna radiation patterns there are used not two separate antennas but one, the antenna radiation pattern of which is periodically switched, occupying alternately positions 1 and 2 (Fig. 4.5). In each of these positions of antenna radiation pattern identical time is found.

For a determination of the direction of deviation of the rocket from the equisignal zone (sign of error ε) it is necessary to distinguish radio signals radiated by the transmitter at positions 1 and 2 of the antenna radiation pattern. For this the signals of the transmitter corresponding to each of the two diagrams, modulate by different laws. For example, in one of the systems of lateral radio-correction [4] at the first position of the antenna radiation pattern the ground station transmits a signal modulated by audio-frequency F_1, and the second position — a signal modulated by frequency F_2. Then in the absence of error ε both signals at the output of the flight receiver of the rocket will be equal in amplitude (Fig. 4.8a); with deviation of the rocket to the left the signal of frequency F_1 will predominate (Fig. 4.8b); with deviation to the right, the frequency F_2 (Fig. 4.8c). With small values of error ε give difference in levels of signals on Fig. 4.8b, c will be proportional to the quantity ε.

For the flight control of ballistic missiles we take an antenna with antenna radiation pattern narrow in the horizontal plane but quite wide in

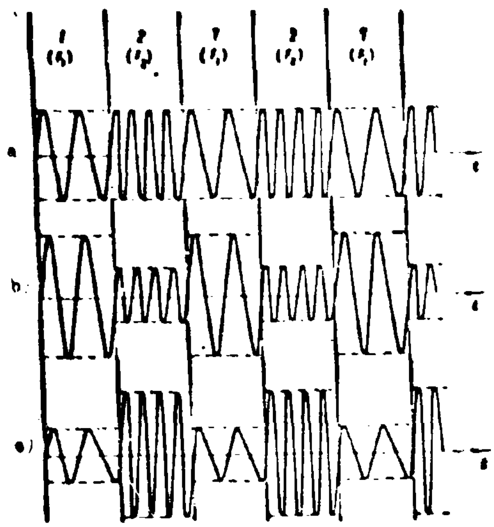

Fig. 4.8. Signals at output of flight receiver: a) during flight of the rocket in the equisignal zone; b) during deviation of the rocket to the left of the equisignal zone; c) during deviation of the rocket to the right of the equisignal zone.

the vertical plane, since the rocket should in lifting upwards in the powered-flight trajectory all the time to be in the zone of irradiation of the transmitter. With this the middle of the equisignal zone has the form of a vertical plane, which before launching of the rocket is combined with the guiding plane. Thus in systems of lateral radio-correction the control station and starting position of the rocket are placed in the plane of guiding on line AB drawn on the earth's surface (Fig. 4.9). On this line (if one were not to consider the rotation of Earth) there is the target.

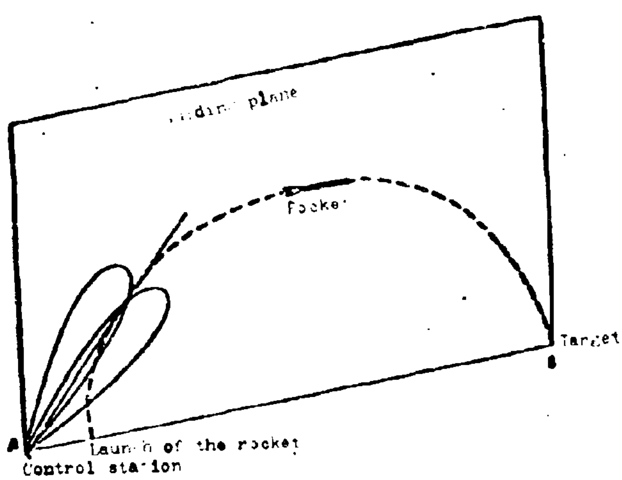

Fig. 4.9. Location of the control station.

Accurate matching of the middle of the equisignal zone with the plane of guiding is fulfilled with the help of a control (adjustment) apparatus. It is placed at a certain distance from the control station in such a manner

that the plane of assigned trajectory of the rocket passes through the antenna of the transmitter and the adjustment apparatus (Fig. 4.10). After that the equisignal zone of the transmitter is guided at the control apparatus. Adjustment

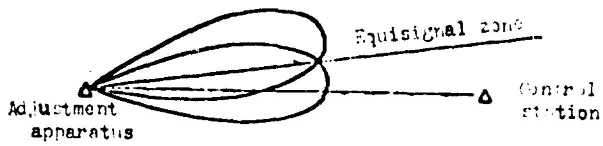

Fig. 4.10. Adjustment of the antenna of the control station.

of the antenna of the control station is a very responsible operation, upon which to a large degree depends the accuracy of the hit of the target. With the adjustment there can appear errors because of inaccurate apparatus of the control station and an inaccurate matching with it of the equisignal zone. Both these errors should be small. A similar adjustment of antennas is produced not only in systems of lateral radio-correction but also in other accurate electronic systems intended for the determination of directions [19].

The ground station of the system of lateral radio-correction includes the antenna, transmitter, timer, switch of antenna radiation pattern and switch of modulating oscillations (Fig. 4.11).

The design of the antenna and the method of switching of the antenna radiation pattern depend on the range radio waves in which the system of correction

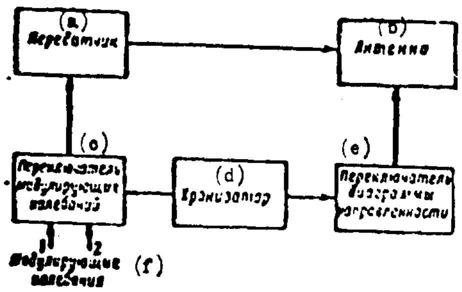

Fig. 4.11. Block diagram of the control station.
KEY: (a) Transmitter; (b) Antenna; (c) Switch of modulating oscillations; (d) Timer; (e) Switch of antenna radiation pattern; (f) modulating oscillations.

operates for example, in equipment of the meter range of waves the antenna system constituted two dipoles located symmetrically with respect to the guiding plane. For switching of the antenna radiation pattern the current of high frequency feeding one of the dipoles was abruptly changed in phase. In the range of shorter waves there can be used less bulky antennas equipped reflectors [4].

The equisignal zone created by the antenna array can be distorted because of the reflection of radio waves from Earth.

Radio waves, being reflected from the earth's surface and being superposed on the basic flux of electromagnetic energy radiated by the antenna, can distort the antenna radiation pattern. The result of this will be a change in the position of the equisignal zone, its slope, deformation and, in the end, the

lowering of the accuracy of the hitting of the target. To avoid these phenomena it is necessary to select a site specially for the placing of the antenna and to take a number of measures to eliminate the influence of the reflected radio waves.

The timer (Fig. 4.11) is an element of the station providing simultaneity of switching of the antenna radiation pattern of the antenna and commutation of the modulating oscillation. If the radio signal of the transmitter is modulated alternately by two audio-frequencies, then the frequency shift of modulation (F_1 and F_2) should occur simultaneously with a change in position of the radiation pattern antenna. The timer controls the operation of elements of the station, which switch the modulating signals and change the position of the antenna radiation pattern.

The control station is placed at a considerable distance from the launching site of the rocket. In one of the control systems described in literature, the distance between the control station and launching site of the rocket was 12 km [4]. The electrical circuits and construction of the control station can differ by great diversity.

A block diagram of the flight radio equipment of the rocket with correction of its flight with the help of the equisignal zone is depicted on Fig. 4.12.

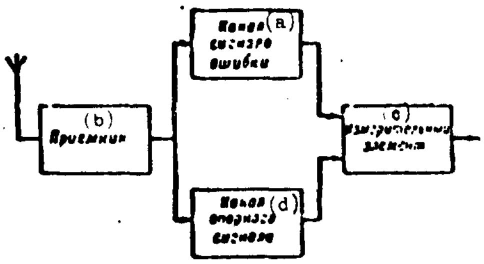

Fig. 4.12. Block diagram of flight radio equipment of the rocket.
KEY: (a) Channel of the error signal; (b) Receiver; (c) Measuring element; (d) Channel of reference signal.

From radio signals accepted on the rocket there are distinguished the error signal and the reference signal. The error signal contains information on the magnitude and direction of deviation of the rocket from the equisignal zone. The reference signal gives information as to which of the two possible positions in each instant is the antenna radiation pattern of the transmitting antenna. This information is needed for the determination of the direction of deviation of the rocket from the guiding plane.

The flight radio equipment has two channels which are intended for the separation of the error signal and reference signal. Further, these signals go to the measuring element (phase discriminator), which forms the signal of lateral correction. This will be discussed in more detail below.

Performance of the circuit of the flight radio equipment of the rocket depends on operating conditions of the transmitter of the ground station (continuous or pulse radiation) and on the method of modulation of radio signals. For example, in the case when the ground station transmits a signal of two audio frequencies F_1 and F_2 the circuit of the flight equipment has the following

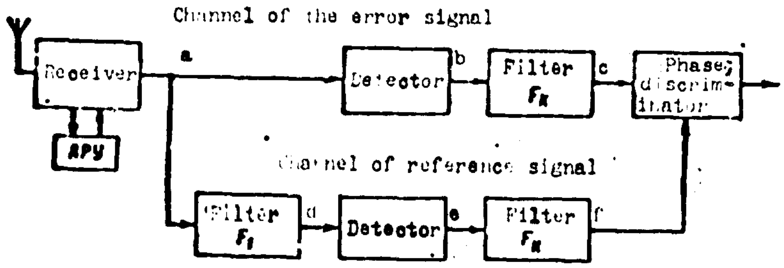

Fig. 4.13. Flight radio equipment of the system of lateral correction for the case when the signal is modulated by two audiofrequencies.

form (Fig. 4.13). It includes a receiver with an automatic amplification control (automatic volume control [AGC] (АПУ), detectors, and filters tuned to the frequency of modulation F_1 (or F_2) and to the frequency of commutation of the antenna radiation pattern F_K. The output element of the equipment is the discriminator. For an explanation of the operation of the circuit containing Fig. 4.14 shows oscillograms of voltages at its points noted on Fig. 4.13 and 4.14 by identical letters.

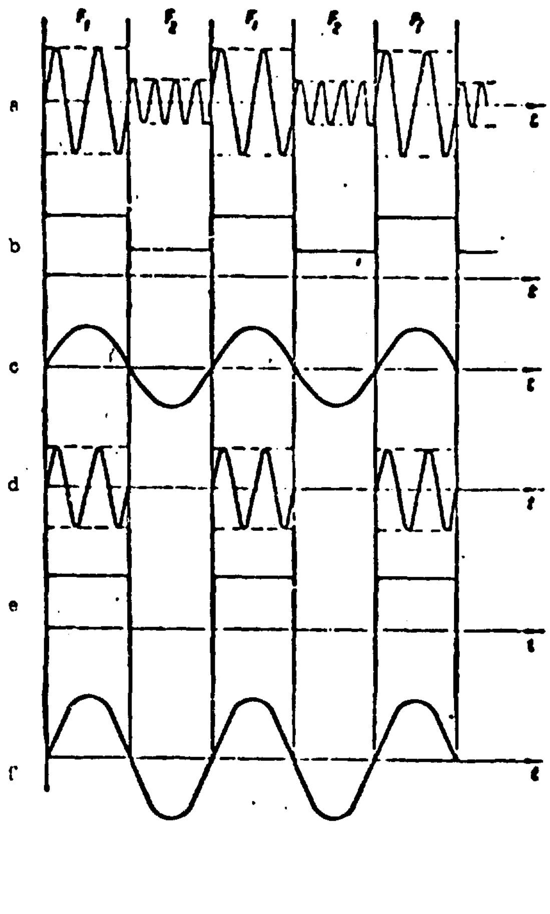

Fig. 4.14. Oscillograms of voltages for the circuit depicted on Fig. 4.13: a) at the output of the receiver; b) after the detector of the error signal; c) at the output of the filter of frequency F_K (error signal); d) at the output of the filter of frequency F_1; e) after the detector of the reference signal; f) at the output of the filter of frequency F_2 (reference signal).

The first oscillogram (Fig. 4.14a) depicts the signal at the output of the flight receiver. The signal constitutes segments periodically repeated through the time interval $\frac{1}{F_K}$ of harmonic frequency variations F_1 or F_2. On

Fig. 4.14 the frequency variations F_1 have larger amplitude than that of frequency variation F_2. This means that the rocket deviated to the left of the plane of guiding, since it was stipulated that the signal of frequency F_1 is radiated at the left position of the antenna radiation pattern of antenna (Fig. 4.5). In one of the control systems [4] there were selected the following values of frequencies; F_1 = 5 kHz, F_2 = 8 kHz; F_K = 50 Hz.

The output signal of the receiver passes along two channels: the channel of separation of error signal and channel of separation of the reference signal (Fig. 4.13). In the first of these channels the signal is fed to the detector, which separates envelope (Fig. 4.14b). Then with help of a filter there is obtained the first harmonic of the envelope with the frequency of commutation F_K (Fig. 4.14c). The voltage depicted on this oscillogram is the error voltage. This voltage is absent while the rocket accomplishes flight in the equisignal zone (i.e., when the error $\varepsilon = 0$). It appears with the appearance of the error, and the value of the error and amplitude of the error voltage are connected by proportional dependence. The direction of deviation of the rocket from the equisignal zone determines the initial phase of the error voltage; if the rocket deviated to the right, the initial phase of sinusoid on Fig. 4.14c would be changed by 180°.

In order to determine the initial phase of the error voltage, it is necessary to compare the error signal with another voltage of the same frequency, the initial phase of which is taken for zero, the reference voltage. With the determination of the position of the rocket relative to the equisignal zone the phase of the reference voltage should be connected with the switching of antenna radiation pattern of the transmitting antenna creating the equisignal zone.

The reference voltage is formed in the following way. With the help of a filter there are distinguished oscillations of only one of the frequencies, for example F_1 (Fig. 4.14d). Then these oscillations are detected and pass through the filter of frequency F_K. As a result there are obtained the envelope (Fig. 4.14e) and its first harmonic (Fig. 4.14f) — the reference signal. This signal does not depend on the attitude of the rocket, but is determined only by the switching of the antenna radiation pattern of the transmitting antenna. The positive half-way of the reference signal coincides with the time interval during which the lobe of the antenna radiation pattern deviates to the left and the negative half-wave — with the interval when the lobe deviates to the right.

By relationship of phases of the error signal and reference signal we judge the direction of the deviation of the rocket: if both signals coincide in phase, the rocket is more to the left of the equisignal zone; if they are different in phase by 180° this means the rocket deviated to the right. The magnitude and sign of deviation of the rocket from the guiding plane are revealed in the phase discriminator.

§ 4.3. Formation of the Error Signal

Let us assume that the ground station of the system of lateral radio-correction radiates signals alternately modulated by one of two audio-frequencies. Such a form of modulation is not, of course, only possible for systems of lateral radio-correction. But from this example we will obtain relations basically correct for other forms of modulation.

The oscillogram of the signal at the output of the flight receiver of the system of radio correction is shown on Fig. 4.14a. This signal constitutes sinusoidal oscillations of one of two audio-frequencies F_1 of F_2:

$$u = U_1 \sin 2\pi F_1 t,$$

when the antenna radiation pattern of the transmitting antenna is in position 1 (Fig. 4.5), and

$$u = U_2 \sin 2\pi F_2 t,$$

when the directional characteristic is in position 2.

If the rocket flies in the equisignal zone, then amplitudes of signals U_1 and U_2 are identical and are equal to the value U_{cp} (Fig. 4.15). This

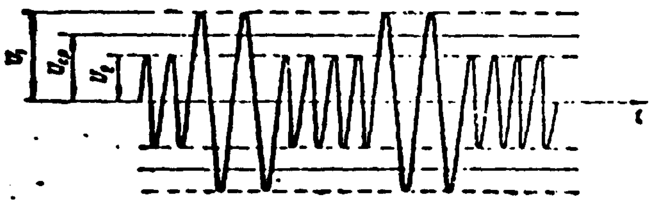

Fig. 4.15. Output signal of the flight radio receiver.

value of the voltage is determined by the amplification factor of circuit from the flight receiving antenna to the output of the receiver, the conditions of propagation of radio waves and the distance between the rocket and ground transmitting station, and also the power radiated by the ground station in the direction of the equisignal zone.

With deviation of the rocket from the equisignal zone of the amplitudes of signals U_1 and U_2 becomes unequal. If, for example, the rocket deviates to the left, the amplitude U_1 increases and U_2 decreases. New values of the amplitude will be equal:

$$U_1 = U_{cp}(1 + m),$$
$$U_2 = U_{cp}(1 - m),$$

where m is the relative increase in amplitude of the signal at the output of the receiver. Changes of both signals as compared to the value U_{cp} can

be considered by identical, but this is true only at small errors ε.

Quantity m determines the modulation percentage of the signal. Modulation percentage depends on how far the rocket deviates from the direction of the equisignal zone. In order to determine the connection between quantity m and error ε, let us examine Fig. 4.16. On this figure there are represented

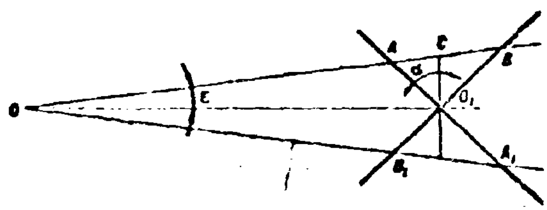

Fig. 4.16. For calculation of the amplitude of the error signal.

two antenna radiation patterns, which form the equisignal zone near the point of their crossing. Antenna radiation patterns for point of crossing O_1 are replaced by straight lines AA_1 and BB_1. For small segments of the curves such a replacement is correct. The angle between rays OO_1 and OB is equal to the error ε.

The amplitude of the signal at the output of the receiver when $\varepsilon = 0$ (i.e., the quantity $U_{\text{ср}}$) is proportional to segment OO_1. Let us take this segment to be unity. Then segments AC and BC will characterize the increase in the signal with the appearance of the error, and since $OO_1 = 1$, then numerically they will be equal to the coefficient of modulation percentage m. For example,

$$m = \frac{AC}{OO_1} = AC.$$

Segments AC and BC on Fig. 4.16 can be considered equal only approximately and at small values of angle ε.

From the drawing (Fig. 4.16) it is clear that

$$\varepsilon = \frac{O_1C}{OO_1} = O_1C.$$

On the other hand

$$AC = O_1C \operatorname{tg} \alpha,$$

consequently,

$$m = \varepsilon \operatorname{tg} \alpha.$$

Here α characterizes the angle at which the antenna radiation patterns cross at point O_1.

Quantity tg α has a fully defined value, depending on the form of the antenna radiation pattern of the antenna. In the last equality this value fulfills the role of the proportionality factor. Therefore, designating

$$k_1 = \operatorname{tg} \alpha. \tag{4.1}$$

we obtain the relation between quantities ε and m in the form

$$m = k_1 \varepsilon. \tag{4.2}$$

The error signal u_C, obtained after the filter of frequency F_K (Fig. 4.14c) is changed according to the law

$$u_\varepsilon = U_\varepsilon \sin 2\pi F_\kappa t$$

or

$$u_\varepsilon = U_\varepsilon \sin (2\pi F_\kappa t + 180°).$$

The first expression pertains to the case when the rocket deviated to the left of the equisignal zone (ε > 0) and $U_1 > U_2$; the second expression is correct for deviation of the rocket to the right (ε < 0), i.e., at $U_1 < U_2$.

The amplitude of the error U_C voltage depends on the modulation percentage of the signal at the output of the receiver, i.e., on the difference between the greatest and least values of the signal;

$$U_\varepsilon = k_2 (U_1 - U_2), \tag{4.3}$$

where k_2 is the transmission factor of the detector and filter of frequency F_K in the channel of the error signal (Fig. 4.13). If one were to recall the dependence (4.2) between the modulation percentage and error value, then it is possible to record

$$U_1 - U_2 = 2U_{cp} m$$

and

$$U_\varepsilon = 2k_1 k_2 U_{cp} \varepsilon. \tag{4.4}$$

The amplitude of the error signal U depends on the error ε. The value of the amplitude of voltage at the output of the receiver U_{cp}, also included in formula (4.4), is a variable. This introduces an uncertainty. In order that by the error signal it be possible to judge simply the magnitude of deviation of the rocket from equisignal zone, it is necessary that the voltage U_{cp} be maintained constant. The problem of maintaining constancy of voltage U_{cp}

is solved by elements of automatic control of amplification of the receiver.

The main factor causing the change in voltage U_{cp} is the increase in distance between the rocket and the ground transmitting station. This distance during the time of operation of the radio equipment of the system of lateral correction can be changed many times. If, however, quantity U_{cp} changes, then at some value of the error ϵ the voltage of the error at the output of the flight radio equipment can have different values, and the deflection of the controls of the rocket can be different. Normal functioning of the control system will be disturbed, and the control of the rocket will be unreliable. Therefore, unfailing and accurate operation of the AGC circuit of the flight receiver is of great importance for the qualitative control of the rocket.

The error voltage with an amplitude determined by relation (4.4), and with an initial phase of 0 or 180° is fed to the phase discriminator (Fig. 4.13). Here the proceeds signal reference, the initial phase of which is constant and is taken to be zero. The task of the phase discriminator is the generation of a signal of direct current, whose value would be proportional to the amplitude of the error signal, but the polarity (positive or negative) would depend on the initial phase of the error signal (0 or 180°).

A large number of various circuits of the phase discriminators, is known but as a basis of all these circuits lies the common principle of operation. Let us examine the simplest circuit discriminator depicted on Fig. 4.17.

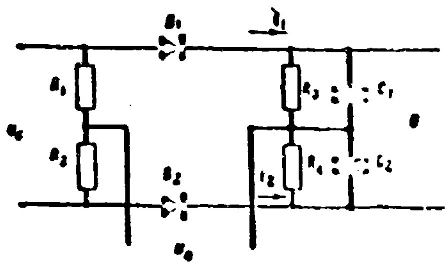

Fig. 4.17. Phase discriminator.

The circuit of the phase discriminator consists of two diodes B_1 and B_2 which are built around separate loads. The load of each diode is the resistor R_3 or R_4 and capacitor C_1 or C_2. The input signal u (error voltage) is fed to the anodes of diodes through the voltage divider R_1 and R_2 and the reference voltage is fed to the median points of the divisor R_1, R_2 and R_3, R_4. Elements included in the circuit of discriminator are selected in such a manner that the circuit be symmetric: diodes B_1 and B_2 should possess identical characteristics, and resistances and capacitances will be selected in such a manner that $R_1 = R_2$, $R_3 = R_4$ and $C_1 = C_2$. For accurate balancing of the discriminators in its circuit there are usually provided adjusting resistances.

The input voltage of the phase discriminator, the error voltage of, can be expressed by the relation

$$u_c' = U_c \sin(2\pi F_0 t + \varphi), \tag{4.5}$$

where the initial phase φ takes one of two possible values $\varphi = 0$ or $\varphi = 180^0$. The amplitude of the input voltage U_c is a variable. It depends on the error ε and is changed from 0 to a certain greatest value of $U_{C,\ max}$. On diodes B_1 and B_2 this voltage enters in reversed phase; on each of the diodes half of the voltage u_c is necessary. The reference voltage is expressed by the dependence

$$u_0 = U_0 \sin 2\pi F_0 t. \tag{4.6}$$

Its amplitude and initial phase are constant. The reference voltage is fed to the diodes in identical phase. Usually the amplitudes of the input and reference signals are selected in such a manner in order to satisfy the inequality

$$U_0 > \frac{U_{c\ max}}{2}. \tag{4.7}$$

The reference voltage is intended for controlling the operation of the phase discriminator.

Figure 4.18 shows oscillograms, explaining the operation of the phase discriminator. On the first group of oscillograms the case is represented when $u_c = 0$, i.e., when the error signal is absent. Currents pass through diodes B_1 and B_2 simultaneously during positive half-waves of reference voltage. During negative half-waves both diodes are closed. In the absence of signal u_c currents i_1 and i_2 flowing in the diodes, are identical, and the output voltage of the discriminator is proportional to the difference $i_1 - i_2$, is equal to zero.

The voltage at the output of the discriminator appears only with the supply of the input signal u_c. The signal u_c can be in phase or in reversed phase with the reference signal. However, if the inequality (4.7) is fulfilled, then in the presence of the input signal both diodes conduct current simultaneously during positive half-periods of the reference voltage. Only the force of currents i_1 and i_2, flowing through the diodes will be changed.

Figure 4.18 shows oscillograms for cases when $\varphi = 0$ and $\varphi = 180^0$. For example, when $\varphi = 0$ to the diode B_1 voltages u_0 and $\frac{u_c}{2}$ are applied in phase and to the diode B_2 — in reversed phase. Consequently, instantaneous values of current i_1 will increase and current i_2 will decrease as compared to the case when $u_C = 0$, and at the output of the circuit there will appear a voltage

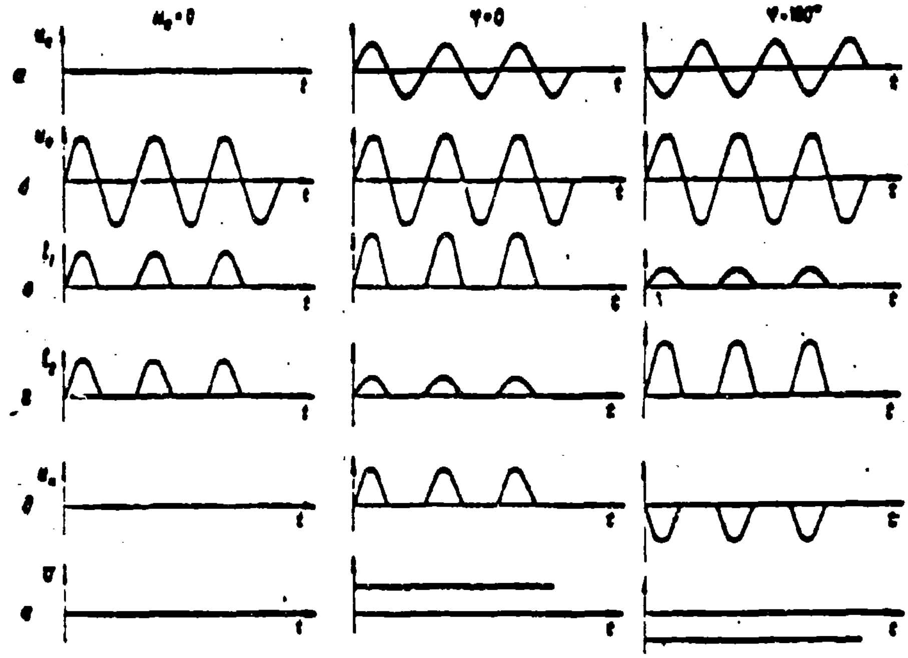

Fig. 4.18. Oscillograms of voltages in the circuit of the phase discriminator: a) error signal; b) reference signal; c) current through diode B_1; d) current through diode B_2; e) voltage at the output neglecting the action of the filter; f) constant component of output voltage.

which will be proportional to the difference of currents flowing in the diodes. Figure 4.18 shows the constant component of output voltage of the discriminator separated with help of the filter R_3C_1 and R_4C_2. With coincidence of initial phases of signals u_C and u_0 the output voltage will be positive. If, however, the initial phase of the error signal $\phi = 180^{\circ}$, then the polarity of the output voltage will be negative.

Rectified voltages on load impedances R_3 and R_4 (let us designate them by U_{R3} and U_{R4}) are proportional to the amplitude of the signal fed to anodes of the diodes;

$$U_{R3} = k_3\left(U_0 + \frac{1}{2}U_c\right),$$
$$U_{R4} = k_3\left(U_0 - \frac{1}{2}U_c\right).$$

where k_3 is the proportionality factor depending on parameters of the diodes and load impedances. The output voltage of the circuit is equal to the difference

$$U = U_{R3} - U_{R4} = \pm k_3 U_c$$

Thus output voltage of the phase discriminator is proportional to the amplitude of the error signal and has a polarity determined by the initial phase of the error signal:

$$U = \pm k_s U_e \qquad (4.8)$$

The plus sign pertains to the case when the error signal coincides in phase with the reference signal, i.e., when in relation (4.5) $\varphi = 0$. Otherwise it is necessary to take a sign minus.

In the flight equipment of the system of lateral radio-correction the circuit of phase discriminator represented on Fig. 4.19 can be used [2].

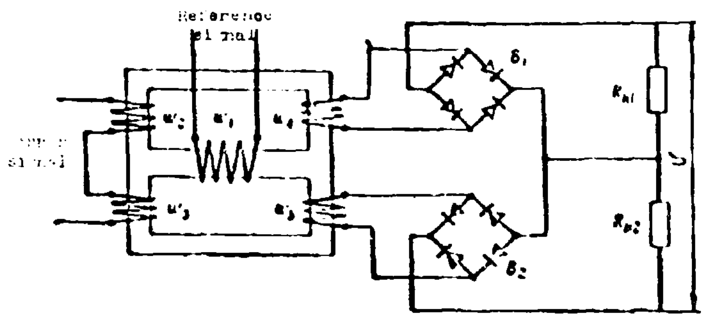

Fig. 4.19. Phase discriminator on a three-core transformer.

This circuit contains a three-core transformer with five windings $w_1 - w_5$. Included in the circuit also are rectifying bridges B_1 and B_2, which produce full-wave rectification of the sinusoidal signal obtained on windings w_4 and w_5.

If the error signal is absent and on the circuit proceeds only the reference signal, then in windings w_4 and w_5 there are induced voltages identical in value. These voltages are rectified in the bridge circuits B_1 and B_2. With the equality of voltages on windings w_4 and w_5 the output voltage of the discriminator U is absent.

The error signal, proceeding to windings w_2 and w_3, can have the initial phase $\varphi = 0$ or $\varphi = 180^\circ$. Windings w_4 and w_5 are located on the core in such a way that in one of them the error signal is found in phase with the reference signal and both signals are added. In the other winding these signals are in reversed phase and, therefore, are subtracted from each other. As a result we will obtain that with the initial phase of the error signal $\varphi = 0$ the voltage in winding w_4 is larger than the voltage on winding w_5. If the initial phase of the error signal is changed to 180° then the voltage on winding w_5 will be greater.

The rectified voltage on resistances R_{H1} and R_{H2} is proportional to the difference of voltages on windings w_4 and w_5 and, in the end, is determined

by the relation (4.8). For the circuit (Fig. 4.19) coefficient k_3 owing to full-wave rectification of the signal, will be larger than that for the half-wave circuit.

The relation for the output voltage of the phase discriminator, included at the output of the apparatus of lateral radio-correction, will be recorded uniting equalities (4.4) and (4.8):

$$U = 2k_1k_2k_3U_{cp}\varepsilon. \tag{4.9}$$

The polarity of voltage U can be positive or negative depending upon the sign of the error ε.

The outputal voltage of the phase discriminator is proportional to the angle ε, characterizing the drift of the rocket from the plane of guiding z. The relation between values ε and z is seen From Fig. 4.20:

$$z = D\,\mathrm{tg}\,\varepsilon \approx D\varepsilon, \tag{4.10}$$

where D is the distance between the rocket and station of the control. The inconvenience of the control of the rocket on the basis of the measurement

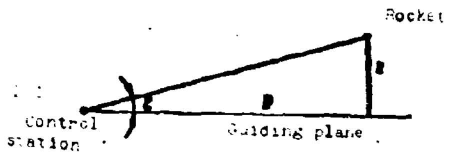

Fig. 4.20. Relation between quantities ε and z, characterizing deviation of the rocket from the guiding plane.

of angle ε instead of the measurement of the most lateral deviation z consists in the fact that the same angle ε corresponds to distinguished values of the lateral deviation z at various distances D.

The voltage U in formula (4.10) is maintained constant owing to the AGC of the receiver. The remaining coefficients included in the formula are constant. Therefore, having denoted

$$k_{p.k} = 2k_1k_2k_3U_{cp} \tag{4.11}$$

we write the simpler relation

$$U = k_{p.k}\varepsilon. \tag{4.12}$$

The signal (4.12) is used for eliminating the drift of the rocket from the plane of guiding.

§ 4.4. Control of Yawing Motion of the Rocket

It is possible to eliminate drift of the rocket from plane of guiding by changing its course, i.e., by means of the influence on the variable Φ. Therefore, the output signal of the system of lateral radio-correction expressed by the relation (4.12) is introduced into automatic machine of stabilization in the control channel of the rocket for yaw. Preliminary voltage (4.12) for improvement of the quality of control of the rocket is subjected to differentiation. Here there can be used the same differentiating RC-circuits as those which were described in the preceding chapter. After that the signals, proceeding from the pitch gyro and from the flight electronic equipment, are summed, amplified and fed to the actuating drive of the controls controlling the course of the rocket (Fig. 4.21).

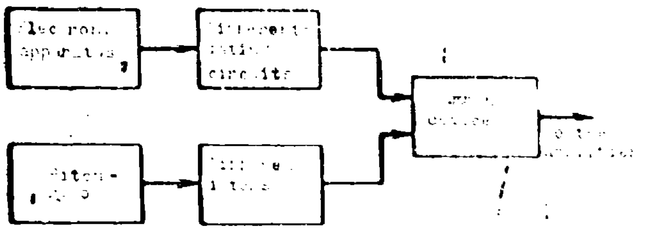

Fig. 4.21. Summing of the signal of yaw stabilization of the rocket and the signal of lateral radio-correction.

Figure 4.22 shows one of the possible circuits of the introduction of the signal of lateral radio-correction into the circuit of automatic stabilization

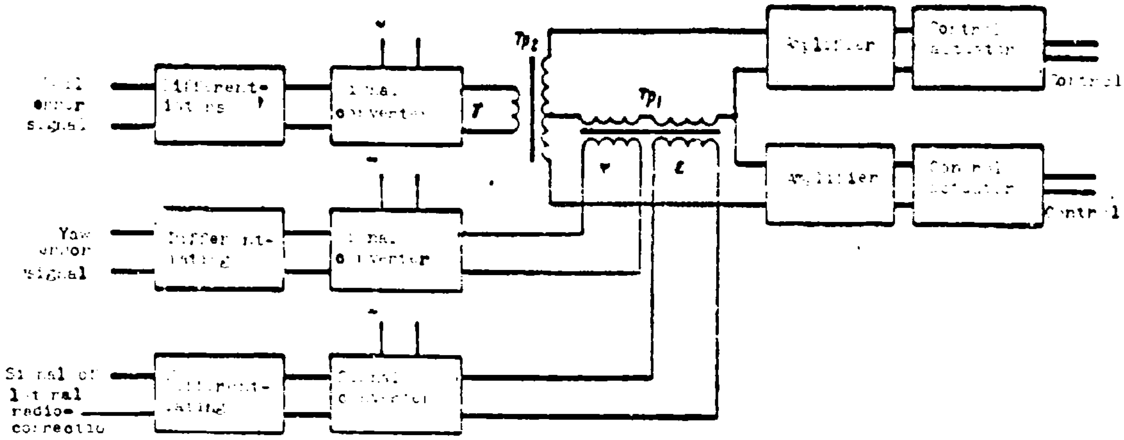

Fig. 4.22. Diagram of the introduction of a signal of lateral radio-correction into the channel of yaw stabilization of the automatic machine.

machine. Elimination of lateral displacement of the rocket from the plane of guiding is carried out by controls I and III of the rocket (Fig. 3.9). To amplifiers of controls I and III proceed signals from the pitch gyro (depending

of errors ϕ and γ) and the signal of lateral radio-correction. With the help of transformers Tp_1 and Tp_2 these signals are added.

The summing is carried out in the following way. The signals are slowly variable voltages. Preliminarily before their summing they are converted into voltages of alternating current. Conversion consists in that these signals modulate the oscillations of the carrying (sound) frequency. Then the amplitude of the carrier oscillation is found proportional to the initial signal, and the change in polarity of the initial signal will be converted into a change by 180° of the initial phase of the carrier oscillation (Fig. 4.23).

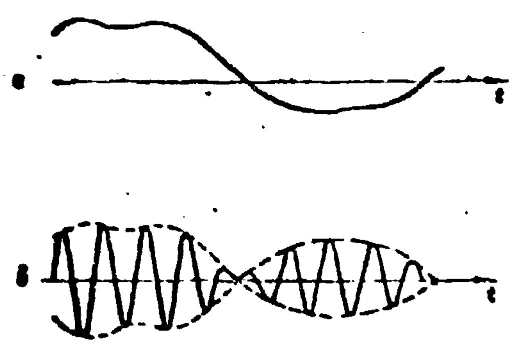

Fig. 4.23. The conversion of slowly variable voltage into modulated oscillations of alternating current: a) initial signal; b) modulated signal.

Algebraic summation of the converted signals is carried out on transformers with several primary windings (Fig. 4.24). For the addition of two signals it is necessary to both primary windings of the transformer in phase. The amplitude of the voltage on the secondary windings U will be proportional

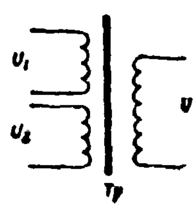

Fig. 4.24. Summing of voltages with the help of transformer.

to the sum of amplitudes of signals U_1 and U_2. For subtraction of the signals they must be fed to the primary windings in reversed phase.

Transformer Tp_1 (Fig. 4.22) is for the summation of signals determined by the yaw error ϕ and lateral displacement of the rocket ε. The signal depending on the roll of the rocket γ, is introduced through transformer Tp_2. Transformer Tp_2 is included so that into the amplifiers of the control actuators I and III the roll signal enters in reversed phase. This means that the signal will cause deviation of the control actuators I and III in opposite directions

and create the moment turning the rocket relative to its longitudinal axis and eliminating roll.

If the differentiators in the channels of yaw ϕ and the lateral deviation ε produce two first derivatives of the signal, then the controls voltage $u_{\text{упр}\,1}$ and $u_{\text{упр}\,2}$ can be written in the following way:

$$u_{\text{упр}\,1} = a_0\psi + a_1\frac{d\psi}{dt} + a_2\frac{d^2\psi}{dt^2}, \tag{4.13}$$

$$u_{\text{упр}\,2} = b_0\varepsilon + b_1\frac{d\varepsilon}{dt} + b_2\frac{d^2\varepsilon}{dt^2}. \tag{4.14}$$

Proportionality factors in formulas (4.13) and (4.14) depend on electrical parameters of the differentiators. The summing controlling signal

$$u_{\text{упр}} = c_1 u_{\text{упр}\,1} + c_2 u_{\text{упр}\,2} \tag{4.15}$$

is fed to the control actuator deflecting the controls of the rocket. Coefficients c_1 and c_2 determine the relationship in which both signals are summed.

Elements of the equipment of lateral radio-correction of the rocket and equipment of the stabilization of the position of its axes together with the rocket itself form a closed automatic control system. This system has two circuits (Fig. 4.25): control circuit of yaw ϕ and control circuit of lateral

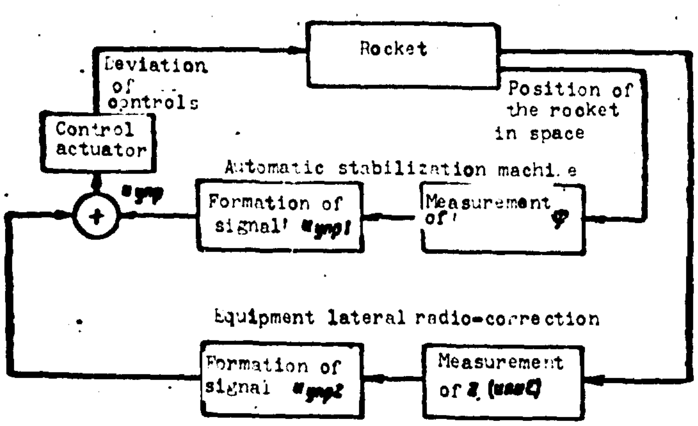

Fig. 4.25. Automatic control system of the course of the rocket and its lateral displacement z (or angle ε).

displacement of the rocket z (or angle ε). The first circuit consists of elements of the automatic machine of stabilization and the second – equipment of lateral radio-correction of motion of the rocket.

It is possible to show that owing to signals of lateral correction, deviation of the rocket from the plane of guiding is limited in magnitude. The stabilized error $z(\infty)$ can be decreased by means of additional amplification of the signal of lateral correction. The transition process appearing in the control of the rocket can also be made quite favorable by selecting properly the level

signals proportional to the first and second derivative of the dirft of the rocket. Basically all positions which were expressed in Chapter 3 in the investigation of the system of stabilization of axes of the rocket are valid here.

Systems of lateral radio-correction of the motion of rockets described in this chapter can be used for the guidance of ballistic missiles of only short range. For long-range rockets they do not provide sufficient accuracy.

As was already noted above in § 4.1, near the guiding plane there is formed a dead zone inside of which the electronic equipment cannot reveal the lateral error ε. The dead zone (equisignal zone) has the form of the sector, and its width determines the minimum angular error $\varepsilon_{\text{мин}}$ which can be revealed by the radio correction equipment. The linear error $z_{\text{мин}}$ is expressed by the relation

$$z_{\text{мин}} = D\varepsilon_{\text{мин}} \tag{4.16}$$

where D is the distance between the rocket and the control station. From formula (4.16) it is clear the the greater the minimum linear deviation of the rocket from the guiding plane, revealed in the system of lateral radio-correction, the greater the distance D. Meanwhile in control systems of lateral motion of ballistic missiles it is most important to attain high accuracy of measurement of the error at the end of the powered-flight trajectory, i.e., at the maximum distance D.

The merit of systems of lateral radio-correction of ballistic missiles is in the relative simplicity of the flight equipment of the rocket and ground control station.

CHAPTER 5

MEASUREMENT OF COORDINATES OF THE ROCKET IN COMMAND SYSTEMS OF RADIO CONTROL

§ 5.1. Determining the Position of the Rocket

The accuracy of guidance of ballistic missiles should be very high. Despite the fact that destructive force of charges of nose cones of the rockets is all the time increasings requirements for the accuracy of hitting a target not only do not decrease, but even increase. This is connected with the fact that protection of strategic objects continuously improves. For example bases of ballistic missiles with underground launchers can withstand a blast wave with a maximum excess pressure equal to several kg/cm^2. If, for example, the pressure at which there is calculated the protection of an object is equal to 7 kg/cm^2 then the destruction of this object with a probability of 90% can be attained with a charge with a TNT equivalent of 5 Mt in the case when the probable radial deviation of the point of fall of the nose cone of the rocket from the target will not exceed 1.8 km [42]. It is known that there have been developed designs with greater protection from the blast wave intended for excess pressure up to 20 kg/cm^2 [3]. For the destruction of such objects there is required even higher accuracy, i.e., almost a direct hit on the target. Therefore, with the development of control systems of ballistic missiles, especially for rockets of long range, we try to use methods and technical means of measurement of parameters of motion of the rocket and generation of commands providing the greatest accuracy.

The measuring complex of the radio control command system includes equipment which measures the coordinates and speed of the rocket on a powered-flight trajectory. Coordinates of the rocket are determined with respect to points on the earth's surface at which there are stations of the radio system (its antenna). Depending upon the principle of operation of the radio electronic system with its help there can be measured the following quantities: angles, distances (ranges), sums of distances, and differences of distances. In accordance with this the systems are called goniometrical, range-finding, total range-finding and difference range-finding.

In range finding system, for determination of spatial coordinates of the rocket we use three basic points on the earth's surface, at which there is set radiotechnical equipment measuring the distances D_1, D_2 and D_3 (Fig. 5.1). If one were to determine the sphere whose center coincides with one of the basic points, and the radius is equal to the distance from the basic point to the rocket, then the position of the rocket in space will correspond to one of the points of the surface of the sphere. Measurements from three basic points permit determining the three spheres whose intersecting point (point M, Fig. 5.1) determines the position of the rocket in space.

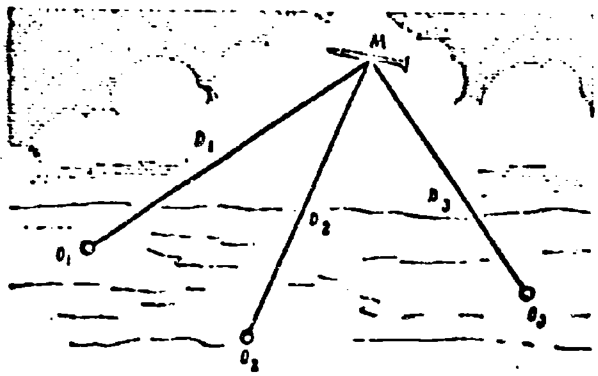

Fig. 5.1. Determination of the point in space by the range-finding system.

In the total range-finding system for the measurement of spatial coordinates we use three bases O_1O_2, O_1O_3, O_1O_4 (Fig. 5.2). On the ends of each base there is located equipment with whose help the sum of distances is measured, for example: $D_1 + D_2$, $D_1 + D_3$ etc. The locus of points satisfying condition $D_1 + D_2 = \text{const}$, is, as is known, a spheroid, the focuses of which will be point O_1 and O_2. Single-values determination of the position of the rocket in space can be attained if one were to determine the three spheroids.

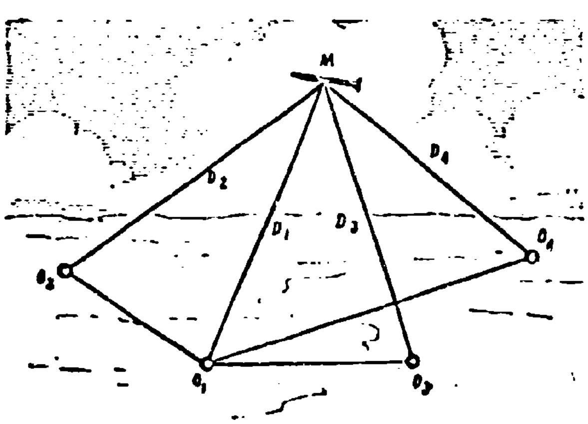

Fig. 5.2. Determining the position of the point in space by a total range-finding or difference range-finding system.

Figure 5.5. shows a diagram with whose help it is possible to measure the time delay of pulses. Signals u_1, the interval of time between which τ

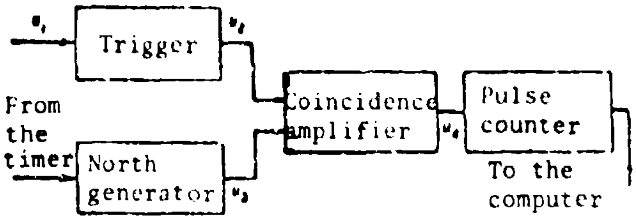

Fig. 5.5. Block diagram of the device for measuring time intervals between pulse signals.

must be measured (Fig. 5.6), proceed to the trigger — a device forming a square pulse of voltage u_2 equal in length to time τ. Voltage u_2 proceeds to the

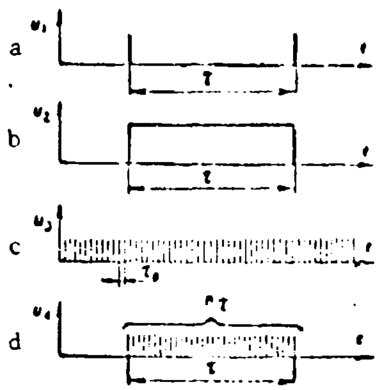

Fig. 5.6. Oscillograms of processes in the device of the measurement of time intervals: a) pulse signals, between which the time interval is measured; b) pulse, produced by a trigger; c) pulses of the generator of calibrated marks; d) signal at the output of the coincidence amplifier.

coincidence amplifier. To this amplifier is also fed voltage u_3 from the marking generator constituting pulses following with a strictly constant period τ_0. The quantity of the period τ_0 determines the error of measurements, and therefore we try to select τ_0 as small as possible. Operation of the coincidence amplifier is characterized by the fact that the voltage u_4 at its output will be formed only with simultaneous existence at the input of voltages u_2 and u_3. The group thus formed of pulse marks proceeds to the counter, in which the number of pulses in group n_τ is determined. The measured time interval τ will be equal to

$$\tau = n_\tau \tau_0$$

number n_τ in a form convenient for subsequent treatment (for example, in binary code) proceeds to the computer of the control system.

With continuous radiation of signals the range is determined also by the time of their delay according to the formula (5.1) or (5.3), but time τ is measured either by the difference of phases of radiated and received waves, or by the difference of frequencies if frequency modulated signals are radiated.

Phase Method of the Measurement of Distance

For the same reasons which were given in the description of the pulse method and during phase measurements of range use we re-emitted signals. The time shift τ between high-frequency oscillations e_1, radiated from the measuring point, and signal e_2, returning from the rocket, will lead to a difference in phase of these oscillations (Fig. 5.7). The phase shift can

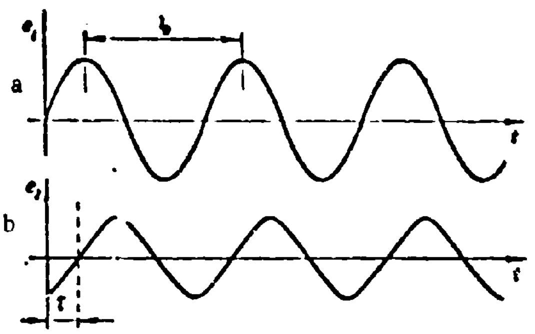

Fig. 5.7. Time shift between signals: a) high-frequency oscillations radiated from the measuring point; b) signal returned from the rocket.

easily be determined from he following considerations. During the time equal to the period of high-frequency oscillations $T_0 = \frac{1}{f_0}$ (where f_0 is the frequency of oscillations), the phase is changed by 2π. Consequently, during the time τ this change will be

$$\varphi = \frac{2\pi}{T_0}\tau. \qquad (5.4)$$

By measuring difference in phases, one can determine the time lag of one signal relative to another and, consequently, also the distance between the measuring point and the rocket.

According to relations (5.3) and (5.4) the relation between the difference of phases ϕ and distance D can be obtained:

$$D = \frac{cT_0}{4\pi}\varphi. \qquad (5.5)$$

If one were to characterize high-frequency oscillations by the wavelength $\lambda_0 = cT_0$, then it is possible to write:

$$D = \frac{\lambda_0}{4\pi}\varphi. \qquad (5.6)$$

From the expression (5.6) it is clear that the phase method permits obtaining great accuracy of measurements if one were to use a radio wave of quite high frequency. Thus, for example, when f_0 = 100 MHz (wavelength λ_0 = 3 m), even with measurement of the difference in phases with not very high accuracy, for example, with an error $\Delta\phi = \pi/8$, the error in the measurement of distance will be less than 10 cm. At so high an accuracy of measurements it is necessary to consider phenomena with which during measurements with errors at several meters or tens of meters it was possible not to consider. Here there belongs the inconstancy of the propagation velocity of radio waves depending upon the state of the medium in which passes the radio link, distortion of the radio beam, etc.

The high accuracy of phase measurements is accompanied, however, by ambiguity of reading. This is connected with the fact that with a change in the distance by a magnitude equal to $\frac{\lambda_0}{2}$ there occurs a full cycle of the change in phase by 2π. Elimination of the ambiguity of reading has known difficulties. There exist two basic methods of the solution of this problem.

In the first method from the beginning of the movement of the object, prior to which distance is measured, counting of cycles of the full change in phase is produced. In this case the measured distance will be equal to

$$D = \frac{\lambda_0}{2} n_\pi + \frac{\lambda_0}{4\pi}\varphi. \qquad (5.7)$$

where n_π is the number cycles of the full change in difference of phases. A deficiency of this method of elimination of ambiguity is that with the interruption of the coupling between the point of measurement and the rocket the count of the cycles and further measurements will be incorrect is lost.

The other method of the elimination of ambiguity consists in the fact that high-frequency oscillations, on which the system operates, modulate by low-frequency oscillations and measure the difference of phases of low-frequency oscillations. The interval of the single-valued reading at low frequency F_0 will be as many times greater as the frequency F_0 is less than f_0. This conclusion is simple to obtain from the relation (5.6). By proper selection of the frequency of modulating oscillations (as a rule, it is necessary to carry out modulation of the high-frequency signal by several low-frequency oscillations) an accurate and single-values reading can be obtained. This can be called the method of multirange reading.

For the measurement of the difference of phases there can be used the examined circuit (Fig. 5.5). It is necessary only to pass from sinusoidal voltage to pulses. For this the sinusoidal voltage u_1 (Fig. 5.8) is amplified and limited in maximum and minimum, so that a voltage u_1' of almost rectangular

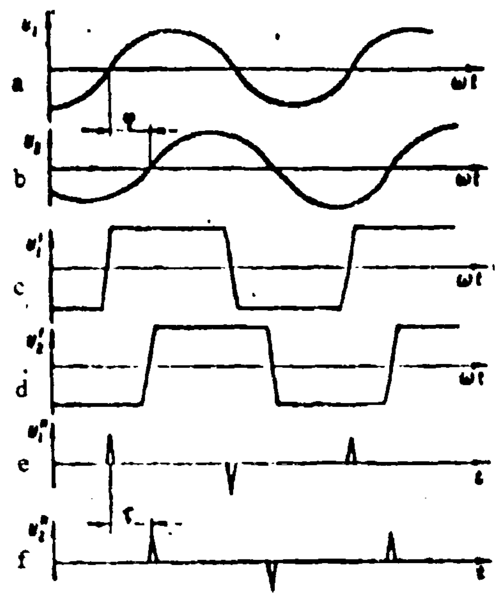

Fig. 5.8. Oscillograms illustrating the transition from the phase shift between harmonic oscillations to a time shift between pulses: a, b) initial signals; c, d) signals after amplifiers-limiters; e, f) pulses at the output of the differentiators.

form will be formed. This voltage will differentiate. At the output of the differentiator at instants corresponding to the passage through zero of voltage u_1, short pulses of u_1'' will be formed. Analogous conversions are fulfilled for voltage u_2. The time interval τ between pulses u_1'' and u_2'' of identical polarity will be proportional to the phase shift φ between voltages u_1 and u_2.

Frequency Method of the Measurement of Distance

In this case from the measuring point frequency-modulated signals are radiated. Frequency modulation can be carried out, for example, by the sawtooth law (Fig. 5.9). With a delay of the re-emitted signal for the time τ between

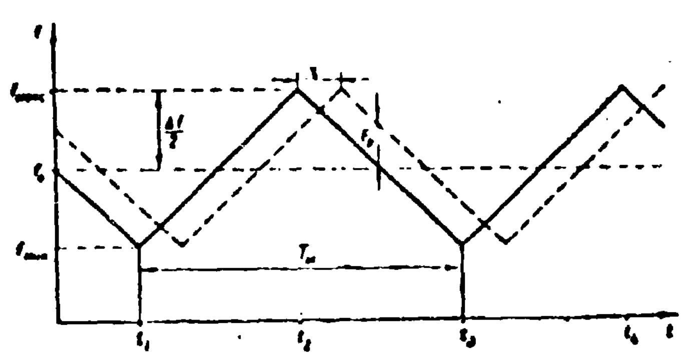

Fig. 5.9. Frequency shift of direct (solid line) and re-emitted (dashed line) signals during measurement of distance in the system with frequency modulation.

oscillations of the ground transmitter and oscillations received from the rocket, there will exist a distinction in frequency equal to F_p. Comparing in the receiving device the direct and re-emitted signals, it is possible to separate the frequency F_p as a beat frequency of two oscillations. It is easy to connect this frequency with the time τ. For the period of frequency modulation $T_M = \frac{1}{F_M}$ Fig. 5.9) high-frequency oscillations will be changed in frequency twice by the magnitude

$$\Delta f = f_{макс} - f_{мин} = 2\,(f_{макс} - f_0),$$

and during the time τ this change will be equal to F_p. Therefore

$$F_p = \frac{4\,(f_{макс} - f_0)}{T_м}\,\tau. \qquad (5.8)$$

The latter expression can be written in the following way:

$$F_p = \frac{2\alpha c F_м}{\lambda_0}\,\tau, \qquad (5.9)$$

where

$$\alpha = \frac{\Delta f}{2 f_0};\quad \Delta f = 2\,(f_{макс} - f_0);\quad \lambda_0 = \frac{c}{f_0}.$$

Considering the relations (5.3) and (5.9), we will find the bond between distance D and difference frequency F_p:

$$D = \frac{\lambda_0}{8\alpha F_м}\,F_p \qquad (5.10$$

In order to estimate what accuracy can be attained with the frequency method of measurement, it is necessary to consider the following circumstances. The voltage of the difference frequency F_p is not a strictly harmonic oscillation. This is connected with the fact that frequency f is changed not monotonically: at time moments t_2, t_4, etc. (Fig. 5.9) the increase in frequency is changed to a decrease, and at moments t_1 and t_3 there occurs an opposite change. Near these points with constancy of the measured distance the difference frequency changes because of a change in character of the frequency modulation. Therefore, a change in difference frequency with a smooth change in distance has an intermitt character, and not all methods of the measurement of frequencies are useful for indication of the frequency F_p. This leads to the fact that it is not possibl to measure frequency F_p by methods convenient in operation if it is less than F_M, and the accuracy of measurements of frequency F_p (error ΔF_p) is also determined by the frequency F_M. Regarding the quantity α determining the depth of the frequency modulation, then, as is known, the achievement of great values of the modulation factor is accompanied by difficulties con ected with the

[illegible] that the transmitting and receiving devices should operate in a broad band of frequencies. Therefore we do not use values of α larger than 0.05-0.1. Taking into account these considerations $\Delta F_p = F_M \alpha = 0.1$, we will obtain for the error of range measurement the quantity $\Delta D = 2.5\ \lambda_0$. Thus if the radio link will operate on quite short waves (centimeter or meter range), then with the frequency method of the measurement of distance high accuracy can be attained.

The approximate composition of the device, with the help of which frequency F_p can be measured, is shown on Fig. 5.10. The voltage of the difference frequency $u(F_p)$ is limited in maximum and minimum and obtains at the output

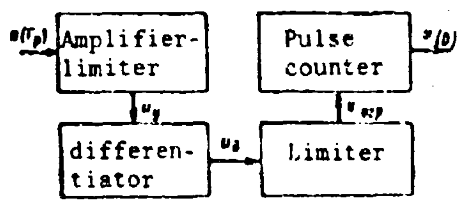

Fig. 5.10. Block diagram of the device for measurement of frequency.

of the amplifier-limiter the form of almost square pulses u_y (Fig. 5.11). From the output of the limiter the voltage proceeds to the differentiator. After differentiation there will be formed short pulses $u_д$ of positive polarity with an increase in voltage u_y and negative with a decrease in the voltage. With the help of minimum limitation negative pulses are filtered, and to the counter proceed pulses $u_{огр}$ of positive polarity.

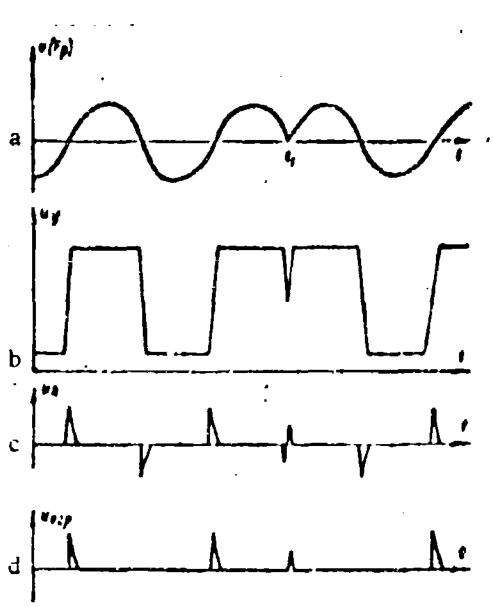

Fig. 5.11. Oscillograms illustrating processes in the device for measurement of frequency: a) voltage of difference frequency; b) signal after the amplifier-limiter; c) pulses at the output of differentiator; d) pulses after the limiter.

In the counter there is formed the signal u(D), proportional to the quantity of pulses entering per unit of time, i.e., proportional to the frequency F_p and, consequently, distance. Oscillograms (Fig. 5.11) show that the disturbance of harmonic character of the change in voltage $u(F_p)$ (point t_1 on Fig. 5.11) is immaterial for the method of measurement used in the device described (if one were to consider that averaging of readings for a sufficiently long time occurs).

Range Systems

The pulse method of the measurement of distance is used in radar stations with whose help parameters of the trajectory of ballistic missiles are determined. Such stations permit determining the position of the rocket with an error of the order of 10^{-5} from distance [30]. With the help of directional antennas the azimuth and angle of elevation of the rocket are also measured.

In one of the systems described in technical literature [33] the position of the rocket in space is determined by the measurement of sums of distances by the pulse method. Making up the system are five ground stations and a flight responder (Fig. 5.12). Station A is the master, and operation of

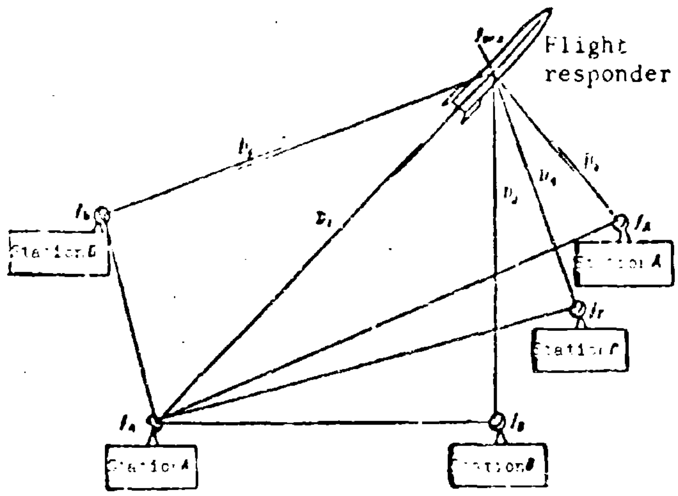

Fig. 5.12. Summing-range system.

system starts with a package by this station of high-frequency pulses (carrier frequency f_A). The flight responder re-emits the pulses, and for improvement of the selection of signals the carrier frequency of the re-emitted pulses F_{OTB} somewhat differs from the frequency f_A.

Signals of the responder are received at all ground stations. Auxiliary stations Б, В, Г, Д in turn re-emit signals of the responder, and each station is at its carrier frequency $f_Б$, $f_В$, $f_Г$, $f_Д$. All the signals enter the master station and owing to different carrier frequencies are clearly distinguished. By the time lag of the signals, knowing the distance between the master station and all the auxiliary stations, we find distances D_1, $D_1 + D_2$, $D_1 + D_3$ etc. This permits determining ellipsoids of possible positions of the rocket. Intersection of the three ellipsoids gives the point in space at which the rocket is found.

The system permits finding more than three (minimum necessary number) ellipsoids. This is caused by the fact that the accuracy of measurements of spatial coordinates of the rocket depends not only on the accuracy of measurements of sums of distances determining every ellipsoid but also on the mutual location of the ellipsoids. Fig. 5.13 shows a case of the determination of a position of a point on a plane when the ellipsoids are replaced ellipses. If the mutual location of ellipses corresponds to Fig. 5.13a, then this favors the high accuracy of the determination of coordinates of point M; if this location corresponds to Fig. 5.13b, then one should expect large errors. Depending upon the region in which the rocket is, we will select three such ellipsoids whose crossing permits obtaining the greatest accuracy of measurements.

There exists a great number of systems operating in conditions of continuous radiation and using the phase method of measurements. In one of the systems [33] three sums of distances are measured — from the central station to the object and from the object to each of the three auxiliary stations. On earth there is installed a transmitter and on the rocket — a responder. For obtaining a single-valved reading with high accuracy of measurements, high-frequency oscillations are modulated by several sinusoidal signals, which serve for phase measurements. Frequencies of the modulation are selected in such a manner that the error of measurements by a coarser scale be clearly less than the interval of the single-valved reading to the nearest more exact scale. Then results of measurements on modulating frequencies give an accurate and single-valved reading of the measured distance.

For a selection of signals of the ground transmitter and signals of the flight responder radio relaying is produced on a carrier frequency higher than the carrier frequency of the ground station.

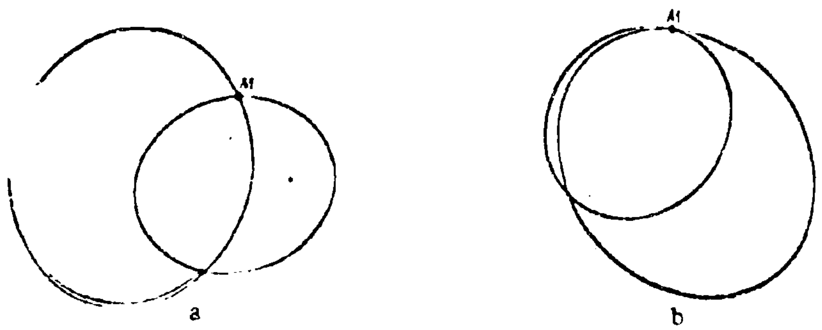

Fig. 5.13. Different cases of mutual location of ellipses: a) favorable location allowing the obtaining of high accuracy of measurement of coordinates; b) unfavorable location.

§ 5.3. Determination of Directions

For a determination of directions in the measuring complex by using polar system of coordinates, measurement is taken of two angles, azimuth angle and angle of elevation of the rocket. Measurement of the angles with help of radio-technical equipment can be accomplished by using directional properties of antennas of receiving or transmitting devices and also on the basis of measurement of the time lag of the signals.

The first method is connected with the measurement or comparison of the force of signals (their amplitude). Such a method of the measurement of angles is called amplitude measurement. To control ballistic missiles amplitude beacons and amplitude direction finders can be used. In the first case directional properties of the antenna of the transmitting device are used. The amplitude beacon is used in the system of lateral radio-correction of the motion of rockets, which is described in Chapter 4. In the second case directional properties of the antenna of the receiving device are used. Amplitude direction finders are used in radio control command systems.

In the determination of direction by the time lag of signals, measurement of the difference in distances from two basic points to the rocket is essentially produced. Figure 5.14 explains this method of measurement. If at points A

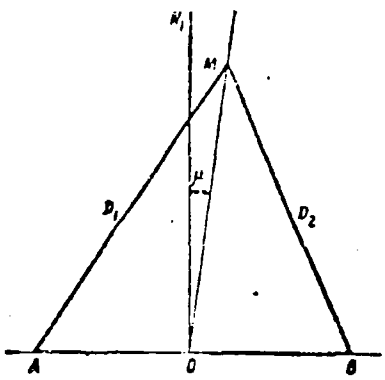

Fig. 5.14. Determination of direction by the time lag of signals.

and B we take the signal from the flight transmitter of the rocket, then under the condition that $D_1 > D_2$ the signal at point A will lag with respect to the signal taken at point B for the time

$$\tau = \frac{D_1 - D_2}{c}. \qquad (5.11)$$

If signals, received at points A and B are observed at the point located at the center of the base (point O) the time interval τ between them will be preserved, since an additional delay of both signals will be identical (distance AO is equal to the distance BO).

By measuring the interval τ we determine the difference of distances

$r = D_1 - D_2$. The locus of points whose difference of distances to two fix points is constant (r = const), will be a hyperbola on the plane. In space the constant difference of distances corresponds to the hyperbolic surface. Thus, the measurement of the time lag of signals permits finding one line of the position of point M, if the plane where this point is found is known or, in general, one surface of the position of point M in space.

If the base of the system (distance between points A and B) is small as compared to the distance from the center of the base to the rocket or the region of possible positions of the rocket is near the normal to the base (line ON_1, Fig. 5.14), then the hyperbolas degenerate into straight lines (on the plane) and the hyperbolic surfaces into conical surfaces. In this case by the difference of distances r we determine the angle μ characterizing the direction at the rocket with respect to the normal to the base plotted in the center of the base. Such a system operates as a goniometrical device.

For the measurement of the interval of time τ in goniometrical devices, as also with the measurement of distances, there can be used pulse, phase and frequency response methods.

Amplitude Direction Finding

In measuring complexes of radio control command systems of ballistic missiles direction finder devices, operating according to the method of the comparison of the amplitude of signals, are used. Direction finding by the method of comparison of amplitudes can be carried out by two methods. In the first case an antenna with a narrow radiation pattern rotates so that the axis of rotation of the main lobe of the pattern (line OO_1 Fig. 5.15) is displaced with respect

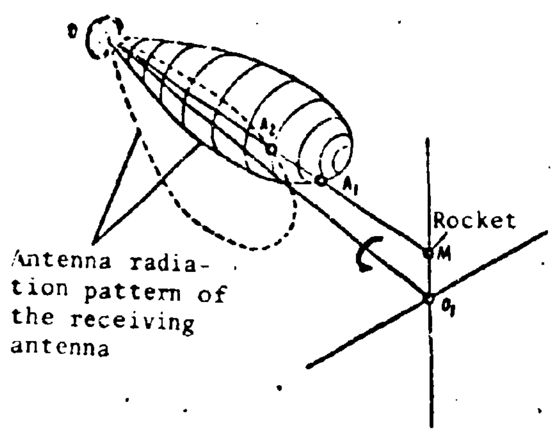

Fig. 5.15. Direction finding according to the method of comparison of amplitudes with a rotating antenna radiation pattern of the receiving antenna.

to the axis of symmetry of the lobe. If the high-frequency signal arrives at the antenna with the direction OO_1, then at any position of the antenna radiation pattern in the process of its rotation around axis OO_1 the intensity

of the reception of the signal will be identical. If the signal arrives from another direction (for example, from the rocket being at point M), then depending upon the position of the radiation pattern of the intensity of reception will be changed: the greatest level of the signal will be proportional to the segment OA_1 and the least level (through a half-turn of the pattern) to segment OA_2. This will lead to modulation of the signal. The frequency of the modulation is equal to the frequency of rotation of the antenna radiation pattern (Fig.

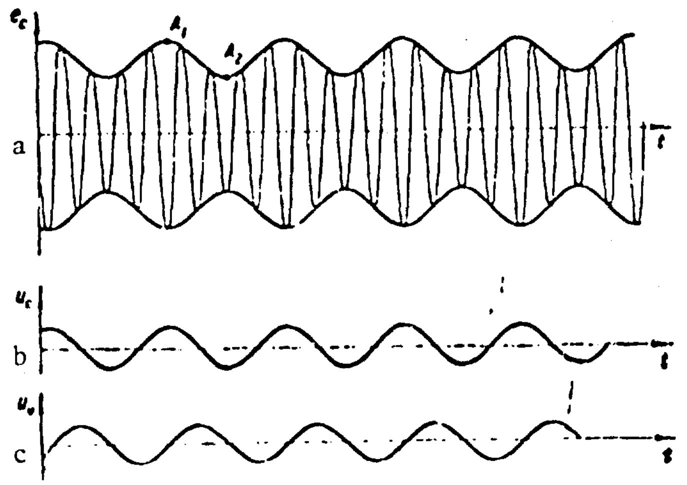

Fig. 5.16. Voltages oscillograms in circuits of the direction finder with a rotating antenna radiation pattern: a) high-frequency signal; b) error signal; c) reference voltage.

5.16), and the initial phase of the envelope of the high-frequency signal depends on the position of the point M, i.e., on the direction of deviation of the rocket from axis OO_1.

After amplification and detection in the receiving device of the high-frequency signal e_c there is isolated the low-frequency voltage u_c (Fig. 5.16). The amplitude and initial phase of this voltage characterize the deviation in space of direction OM from direction OO_1. In order to obtain information on the mutual location in space of directions OM and OO_1, signal u_c (error signal) is compared with the reference voltage u_0, the initial phase of which is constant.

Using voltage u_c as the control, it is possible to turn the antenna system in such a manner that direction OM coincides with direction OO_1. Then the error signal will become equal to zero. The position of axis OO_1 with respect to the design of the antenna is known, and the movement of the antenna relative to the initial position is accurately measured. Therefore, when $u_c = 0$ can be determined the direction to the rocket (azimuth and elevation).

In the second method of direction finding according to method of comparison of amplitudes the antenna radiation pattern does not rotate, but there are used antennas creating four narrow-directional lobes (Fig. 5.17). Two lobes, partially overlapping one another, form an equisignal plane perpendicular to the earth's surface; the other two lobes form an equisignal plane, perpendicular

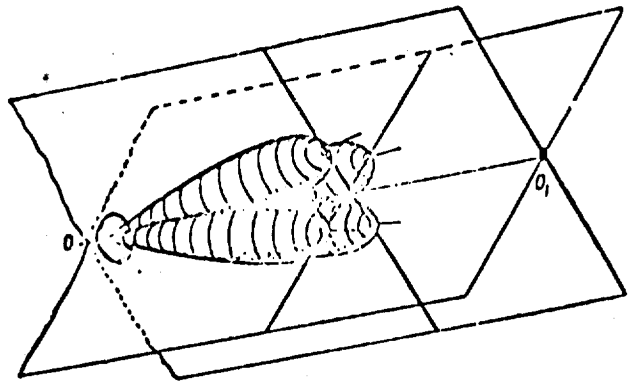

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Fig. 5.17. Direction finding according to the method of comparison of amplitudes with a fixed antenna radiation pattern.

to the first. The intersecing of these planes is an equisignal line for both pairs of antennas. Comparing the signals of each pair of antennas with each other, we obtain a voltage characterizing the attitude of direction-finding object (rocket relative to the equisignal line. With the help of mismatch signals it is possible to move the antenna to such a position when the equisignal line will be directed towards the rocket, which will allow producing measurement of azimuth and elevation.

By comparing both methods of amplitude direction finding, it is possible to state the following. First, in direction finding with the help of a rotating antenna radiation pattern there is required a greater time for the determination of the angular coordinates. Results of the measurement can be obtained only after, at least, one full turn of the antenna radiation pattern. In direction finding by the second method the time necessary for measurements is many times less, since the result can be obtained with the entering from the object of one pulse. Therefore, similar systems are often called, monopulse, although they can operate with continuous signals.

Secondly, the accuracy of measurement of angular coordinates in direction finders with a rotating antenna radiation pattern is affected greater by interferences. This is caused by the fact that with the formation of a mismatch signal in these direction finders there are compared pulse amplitudes coming to the receiving antenna of the direction finder at different instants. The level of interferences in the course of time is changed, they will affect amplitudes of comparable signals differently, and this will lead to the appearance of spurious signal of mismatch and to errors in measurement. In direction finders operating on the second method, pulse amplitudes coming from the object are compared simultaneously. With the comparison (subtraction) of these pulses interferences are mutually compensated.

The accuracy of measurement with the help of amplitude direction finders depends on the perfection of the measuring devices and the signal-to-noise ratio and is determined by the sharpness of the antenna radiation pattern of antenna. With a large signal-to-noise ratio in direction finders with the rotation of the antenna radiation pattern the error of measurement of the angles

can have a magnitude approximately equal to $\Delta\mu \approx 0.02\Theta$, where Θ is the width of the antenna radiation pattern on a level of half-power. In the direction finder with simultaneous comparison of signals it is possible to attain an accuracy at which $\Delta\mu \approx 0.005\,\Theta$ [30]. The angle Θ depends on the effective wavelength of the direction finder λ_0 and geometric dimensions of the antenna. For example, for an antenna in the form of a parabolic mirror

$$\Theta \simeq 70\frac{\lambda_0}{D},$$

where Θ is measured in degrees; D is the diameter of the mirror in the same units as the wavelength. Consequently, with amplitude direction finding

$$\Delta\mu \simeq (0.35 - 1.4)\frac{\lambda_0}{D} \text{ deg}$$

The small magnitude of the error $\Delta\mu$ can be attained (with not too large dimensions of the antennas) only during operation of the direction finder at superhigh frequencies. Then, considering the possibility of unfavorable conditions of the reception of the signals, dimensions of the antenna will be taken larger than the last formula requires.

Determination of Directions by the Time Lag of Signals

The direction the rocket can be determined by the time lag of signals passing the distance from the object to the two basic points (Fig. 5.14). This time τ is equal to r/c, where $r = D_1 - D_2$ is the difference of distances between the basic points and the object. For definitiveness we will assume that signals are radiated from aboard the rocket and are received at two points on earth. Directional properties of antennas with such a method of measurements are used only for the improvement of reception of signals (increase in the signal-to-noise ratio) and for increasing the secrecy of operation of the system.

Let us find the coupling between time τ and direction to the rocket in the base - rocket plane. The angle μ determining this direction is measured, with respect to the normal to the base (Fig. 5.18). In real conditions of

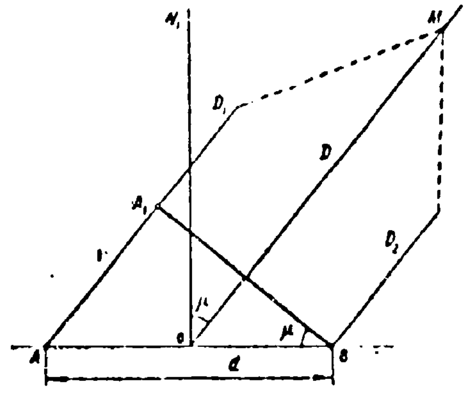

Fig. 5.18. Determination of the relationship between the length of the base, direction to the rocket and time lag of of the signals.

operation of the goniometrical system the distance to the rocket D exceeds many times the dimensions of the base d. In this case it is possible to assume that lines AM, BM and OM are parallel to each other. If from point B we drop a perpendicular to line AM (point A_1), then A_1M = BM and segment AA_1 is equal to r. Angle ABA_1 is equal to angle μ. Consequently, it is possible to write

$$r = d \sin \mu \tag{5.12}$$

and

$$\tau = \frac{d \sin \mu}{c}. \tag{5.13}$$

If line ON_1, which is normal to the base, lies in the guidance plane of the rocket, then deviations of the rocket from this line will be insignificant. Therefore the angle μ will be small even with the largest permissible deviation of the rocket from the assigned trajectory. Then it is possible to take $\sin \mu \doteq \mu$ and write

$$\tau = \frac{d}{c} \mu \tag{5.14}$$

or

$$\mu = \frac{c}{d} \tau. \tag{5.15}$$

By measuring the time lag of signals τ one can determine the direction to the rocket (measure angle μ). This principle can be assumed as a basis of different systems depending upon the method of measurement of time: pulse, phase and frequency. Let us consider these methods of measurement of the quantity τ.

1. Pulse method. In the case of the application of this method the signals have the form of short-duration pulses. By measuring the time between the reception of pulses coming from the rocket at points A and B (Fig. 5.18), we determine the delay of signals τ. These measurements are analogous to those which are produced in pulse range finders, but the error in the measurement of angle depends not only on the accuracy of the measurement of time τ but also on value of base. If angle μ is small and it is possible to consider $\sin \mu \doteq \mu$, then from formula (5.15) we will obtain

$$\Delta\mu = \frac{c}{d} \Delta\tau. \tag{5.16}$$

For example, if the accuracy of the measurement of the time interval is determined by the error $\Delta\tau$, not greater than 0.03 μs, then so that the error of the measurement of angle $\Delta\mu$ does not exceed 1 mrad, the system should have a base d of not less than 9 km. This makes the measuring complex bulky.

The merit of the pulse method of measurements is the fact that with it there are no difficulties connected with the removal of ambiguity of reading.

2. Phase method. The measurement of time τ in phase systems is produced by the difference of phases of two signals received in points A and B (Fig. 5.18). As was shown in § 5.2, the relation between time τ and difference of phases ϕ has the form:

$$\varphi = \frac{2\pi}{T_0} \tau.$$

Considering this relation and carrying out the replacement $\lambda_0 = cT_0$, we will obtain from (5.15) for small angles of μ

$$\mu = \frac{\lambda_0}{2\pi d} \varphi. \tag{5.17}$$

From the latter formula

$$\Delta\mu = \frac{\lambda_0}{2\pi d} \Delta\varphi. \tag{5.18}$$

High accuracy of measurement of angle μ can be obtained by two means. The first means consists in the achievement of high accuracy of measurement of phase shifts with a not very small ratio $\frac{\lambda_0}{d}$. In the second case we have recourse to bases of large value as compared to the length of the operating wave, which permits lowering the requirements, for the accuracy of measurement of difference of phases. By certain means we find the application in practice. In command systems of radio control of rockets there can be selected radio waves of very high carrier frequency f_0. In this case the necessary accuracy of measurements can be obtained owing to the selection of the small magnitude of the ratio $\frac{\lambda_0}{d}$. For example, if λ_0 = 3 cm, base d = 4 m, then even at an error of measurements of difference of phases $\Delta\phi = \frac{\pi}{4}$ the error of determination of direction will not be larger than 1 mrad.

With the phase method of the measurement of angles, just as with the phase method of the measurement of distance, there is the question of the elimination of ambiguity of reading. Actually, in the measurement of angle μ is a magnitude equal to $\frac{\lambda_0}{d}$, there will occur measurement of the phase angle ϕ for full cycle, 2π. To eliminate the ambiguity of reading there can be used methods discussed in § 5.2 and also the method founded on the change in the length of the base. Exact measurements are produced with base d_1, providing a small ratio $\frac{\lambda_0}{d_1}$ i.e., high accuracy. To eliminate the ambiguity we pass to the operation with a smaller base d_2 at which the sector of the single-valued reading has such a magnitude that during flight in a powered-flight trajectory the rocket cannot emerge outside its limits. With this there should be fulfilled the condition that the magnitude of the sector of the single-valued reading with accurate measurement be clearly larger than the error of measurement of angle μ in working with a small base.

3. Frequency method. For realization of the frequency method of the

measurement of angles signals radiated by the flight equipment should be frequency modulated. Frequency modulation can be carried out in the same way as in the frequency range finder (see § 5.2). In accordance with the earlier obtained dependences (5.9) and (5.15), it is possible to write

$$\mu = \frac{\lambda_0}{2 a d F_M} F_p \tag{5.19}$$

We will assume that the error of measurement of the difference frequency F_p is equal to F_M, and the modulation percentage $a = 0.1$ (see § 5.2). In this case for achievement of the error of measurement of angle μ not larger than 1 mrad, it is necessary that $\frac{\lambda_0}{d}$ be equal to or less than $2 \cdot 10^{-4}$. Even for a very high operating frequency $f_0 = 10^{10}$ Hz ($\lambda_0 = 3$ cm) in this case a base 150 m wide is necessary.

Goniometrical Systems

According to data of literature there exists a large number of goniometrical systems intended for the control of trajectories of rockets and artificial earth satellites in which measurements are produced by the phase method. As a rule, these systems are designed in such a way that it is possible to determine both azimuth and the angle of elevation.

Measurement of the phase shift between high-frequency oscillations is strived for at a low frequency; the phase relationships between signals of low frequency should accurately correspond to the difference of phases of high-frequency oscillations. The transition to low frequencies is connected with the fact that technically it is considerably simpler to realize an accurate measuring device for the determination of difference of phases at low frequencies than at superhigh frequencies utilized as operating (carrying) frequencies of the system. An example of how these problems are solved can be the systems described in technical literature.

For the measurement of two coordinates of a direction finding object (azimuth ν and elevation ζ) it is necessary to have two bases. In particular, these bases can be formed by three antennas. One antenna is common for both bases, and the other two antennas are located on mutually-perpendicular lines (Fig. 5.19). For simplicity of account let us assume distances AB_1 and AB_2 to be identical and equal to d. In the receiving-measuring device we determine the phase shifts ϕ_1 and ϕ_2 between the high-frequency oscillations radiated by the flight transmitter and received on earth. To determine ϕ_1 we measure the phase shift of signals between antennas A and B_1, and to determine ϕ_2, between antennas A and B_2.

The base of system d has small dimensions in comparison with the distance to the direction finding object, and therefore radio beams arriving from the object to the antennas can be considered to be in parallel. Under these conditions we can write

$$\varphi_1 = \frac{2\pi d}{\lambda_0} \sin \mu. \tag{5.20}$$

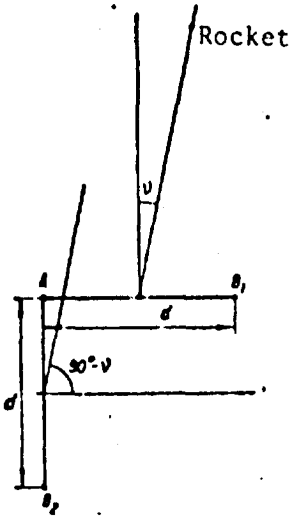

Fig. 5.19. Location of antenna in the phase system of the measurement of azimuth and elevation.

Between angle μ and its projection on the horizontal plane – azimuth ν there exists the following coupling (Fig. 5.3):

$$\sin\mu = \sin\nu\cos\xi, \tag{5.21}$$

where ξ is the angle of elevation. Thus

$$\varphi_1 = \frac{2\pi d}{\lambda_0}\sin\nu\cos\xi. \tag{5.22}$$

The base of the antennas AB_2 is perpendicular to base AB_1, and therefore for phase angle ϕ_2 the following expression will be correct:

$$\varphi_2 = \frac{2\pi d}{\lambda_0}\cos\nu\cos\xi. \tag{5.23}$$

If one were to take the ratio of expressions (5.22) and (5.23), then we obtain

$$\frac{\varphi_1}{\varphi_2} = \mathrm{tg}\,\nu,$$

and, consequently, the azimuth can be determined by the formula

$$\nu = \mathrm{arctg}\,\frac{\varphi_1}{\varphi_2}. \tag{5.24}$$

According to relations (5.22) and (5.23)

$$\varphi_1^2 + \varphi_2^2 = \frac{4\pi^2 d^2}{\lambda_0^2}\cos^2\xi,$$

which makes it possible to determine the angle of elevation

$$\theta = \arccos\left(\frac{\lambda}{2\pi d}\sqrt{\varphi_1^2 + \varphi_2^2}\right). \qquad (5.25)$$

Let us examine now how it is possible to carry out transition from the phase shift between high-frequency oscillations to the phase shift between low frequency signals. The circuit should be built in such a way that additional phase shifts in the equipment do not distort results of the measurements. A block diagram of the system in which there is realized one of the possible solutions of this problem is depicted in Fig. 5.20 [11, 29].

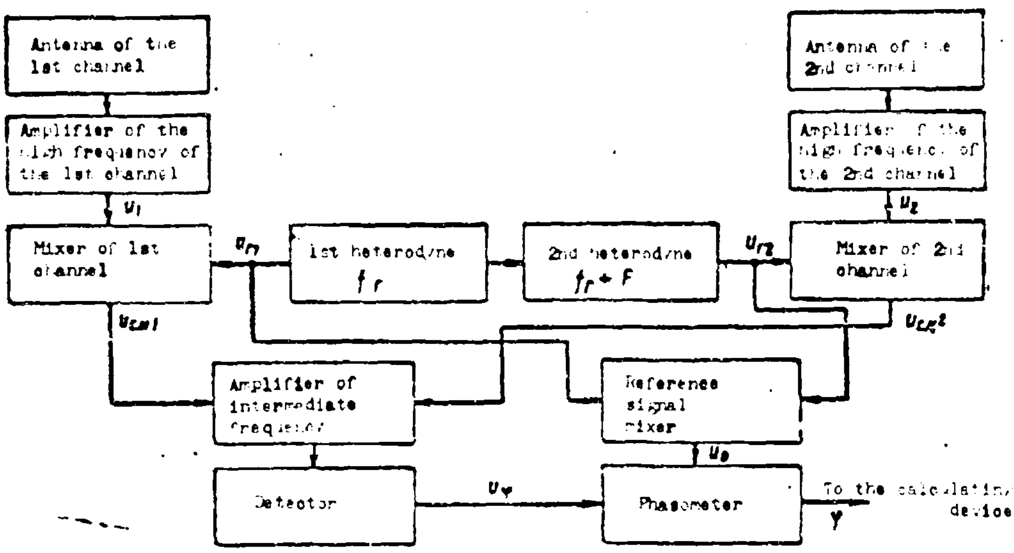

Fig. 5.20. Block diagram of the apparatus measuring the difference of phases between signals received on two spaced antennas.

If the phase shift between signals in antennas of the first and second channels is equal to φ then for the voltage u_1 and u_2 at the output of the high frequency amplifiers it is possible to write:

$$u_1 = U\cos\omega_0 t, \qquad (5.26)$$

$$u_2 = U\cos(\omega_0 t - \varphi). \qquad (5.27)$$

where the angle φ is determined by formula (5.22) or (5.23), and $\omega_0 = 2\pi f_0$, where f_0 is the frequency of the signal of the flight transmitter. In the mixer of the first channel there occurs the addition of signal u_1 and signal of frequency f_r, proceeding from the first heterodyne. The voltage of this

heterodyne can be written in the following way:

$$u_{\Gamma 1} = U_\Gamma \cos(\omega_\Gamma t + \varphi_{\Gamma 1}) \tag{5.28}$$

In the mixer of the second channel there is added the signal u_2 with the signal of the second heterodyne, and the voltage of this heterodyne is obtained from the voltage of the first heterodyne by the increase in its frequency to a certain quantity

$$\Omega = 2\pi F\,(F \ll f_0).$$

Consequently, for the output voltage of the second heterodyne it is possible to write

$$u_{\Gamma 2} = U_\Gamma \cos[(\omega_\Gamma + \Omega)t + \varphi_{\Gamma 1} + \varphi_{\Gamma 2}] \tag{5.29}$$

As a result of the operation of the mixers, at their output with the help of filters there are isolated signals of difference frequencies $\omega_0 - \omega_\Gamma$ and $\omega_0 - \omega_\Gamma - \Omega$, so that for voltages u_{cM1} and u_{cM2} it is possible to write

$$u_{cM1} = U_{cM} \cos[(\omega_0 - \omega_\Gamma)t - \varphi_{\Gamma 1}]. \tag{5.30}$$

$$u_{cM2} = U_{cM} \cos[(\omega_0 - \omega_\Gamma - \Omega)t - \varphi - \varphi_{\Gamma 1} - \varphi_{\Gamma 2}]. \tag{5.31}$$

These signals enter the basic amplifier of intermediate frequency, the passband of which permits without frequency and phase distortions to amplify the signals in a frequency range from $(f_0 - f_\Gamma - F)$ to $(f_0 - f_\Gamma)$. Owing to the proximity of frequency of voltages u_{cM1} and u_{cM2} there are formed beats and at the output of the detector the voltage of the difference frequency will be isolated:

$$u_\varphi = U_\varphi \cos(\Omega t + \varphi + \varphi_{\Gamma 2}). \tag{5.32}$$

All high-frequency components after the detector are filtered.

In order with the help of the phasometer to determine quantity ϕ at the input of the phaseometer, besides voltage u_ϕ, there should proceed the reference voltage u_0 of the frequency $\Omega = 2\pi F$ with the initial phase equal to $\phi_{\Gamma 2}$. For obtaining the voltage u_0 the mixing of signals of the first and second heterodynes is carried out. The voltage at the output of the mixer of the reference signal will be characterized by the expression

$$u_0 = U_0 \cos(\Omega t + \varphi_{\Gamma 2}). \tag{5.33}$$

By comparing formulas (5.32) and (5.33), we see that the difference in phases of the two low-frequency voltages u_{ϕ} and u_0 accurately corresponds to the difference in phases between radio-frequency voltage in antennas of the first and second channels.

A large number of phasometeric goniometrical systems operates on these principles or ones close to them. In one of the systems [30] for accurate determining of angular coordinates a base with a length of 150 m at an operating frequency of ~100 MHz is used. This gives the ratio $\frac{d}{\lambda_0}$ equal 50. The error of measurement of the difference of phases does not exceed $\frac{\pi}{100}$ and as a result the accuracy angular measurements is characterized by an error approximately of 20″. To remove ambiguity of reading there is provided an operation with shortened bases of the length 15 m. To improve conditions of the reception signals directional antennas are used each of which consists of 8-12 horizontal dipoles located at a distance of 1.5-2 m from the ground. Installation of receiving antennas on the terrain is produced with the help of geodesic instrument with a precision of ~6 mm. The frequency at which measurements of the difference of phases are produced is equal 500 Hz.

§ 5.4. Measurement of Speed

In systems of the radio control of rockets, for the formation of command signals it is necessary to have not only data on the position of the rocket in space, but also to know with high accuracy the magnitude and direction of its speed. With the help of radiotechnical means first of all the radial speed is measured, i.e., the speed of the rocket along a line connecting the rocket and the control center. If the direction of the velocity vector does not coincide with the rocket-control center line, then the angles ξ, μ, and ν will be changed. Therefore, in command systems of radio control it is necessity to measure not only the radial but also the angular velocities.

Measurement of Radial Speed

In the preceding paragraphs methods were discussed of the determination of spatial coordinates of the rocket. With continuous measurement of these coordinates it is possible to calculate the velocity component vectors. As is known, to solve such a problem it is possible to compute the time derivatives from functions expressing the change in coordinates. The derivatives will be equal to the components of speed in the direction of the corresponding axes. For example, the value of radial speed $\dot{D}$ (component of speed in the direction of the radius-vector of the polar system of coordinates) is found from the expression

$$\dot{D} = \frac{dD(t)}{dt}. \tag{5.34}$$

Mathematical operation of the determination of the derivative is accomplished with the help of a computer included in the equipment of the command guidance

system. With this, as a rule, the finding of the derivative from function $D(t)$ is replaced by the determination of the finite difference quotient of the function $D(t_2) - D(t_1)$ for the small interv ' of time to this interval $t_2 - t_1$ and expression (5.34) is replaced by 'ormula

$$D_r = \frac{D(t_2) - D(t_1)}{t_2 - t_1}. \tag{5.35}$$

Here $D(t_1)$ and $D(t_2)$ are distances between the point of measurement and rocket at instants t_1 and t_2. With such (indirect) method of the determination of speed, to achieve high accuracy it is necessary to carry out averaging of the results of measurements, which leads to a delay in the obtaining of data necessary for operation of the control system and in the end negatively affects the accuracy of control. Therefore, in command guidance systems of rockets direct methods of the measurement of radial speed, founded on use of the Doppler effect, are used.

The Doppler effect consists in the fact that the frequency of oscillations received by the receiver in the case when the receiver and radiator move relative to each other differs from the frequency of oscillations received by the receiver in the absence of movement. With the approach of the source of oscillations with the receiver there occurs an increase in frequency and in the departure of it, a lowering.

The reason for this phenomenon is simple to explain by examining the radiation and reception of harmonic oscillations. Let us assume that at a certain instant at the point of reception B the phase of harmonic oscillations radiated by the transmitter located at point A corresponds to maximum (Fig. 5.21). If the

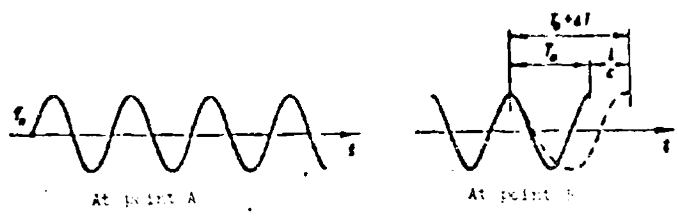

Fig. 5.21. Oscillograms clarifying the Doppler effect.

receiver and transmitter do not move relative to each other, then upon the expiration of time $T_0 = \frac{1}{f_0}$ (where f_0 is the frequency of the radiated oscillations) the phase of oscillations at point B will again correspond to the maximum. In this case the frequency of the received oscillations will be equal to the frequency of the radiated oscillations.

If the receiver will start to move with respect to the transmitter (for

example, departing from it), then the time of arrival to it of the second maximum will be increased as compared to the preceding case by ΔT. The time ΔT is conditioned by the fact that the electromagnetic wave should pass the additional distance ℓ at which the receiver moves during $T_0 + \Delta T$. It is obvious that $\ell = (T_0 + \Delta T)V_p$, where V_p is directed along the transmitter-receiver line, i.e., V_p constitutes the radial speed.*

As a result of movement of the receiver radio waves at point B will be received not as oscillations, with the period T_0, but as oscillations with the period T_1 equal to $T_1 = T_0 + \Delta T$. The quantity ΔT will consist of $\Delta T = \frac{\ell}{c}$, where c is the speed of propagation of electromagnetic oscillations. Considering that $\ell = (T_0 + \Delta T)V_p$, it is possible to write

$$T_1 = \frac{T_0}{1 - \frac{V_p}{c}}. \tag{5.36}$$

Turning to the frequency of oscillations, we will obtain that the frequency f_1 of the signal, which will be received by the receiver in its travel with respect to the transmitter with speed V_p, will be equal to

$$f_1 = f_0\left(1 \mp \frac{V_p}{c}\right). \tag{5.37}$$

The minus sign corresponds to the departure of the receiver from the transmitter and the plus sign, to their approach.

From expression (5.37) it is easy to determine the difference frequency (Doppler frequency) $F_Д$:

$$F_Д = |f_0 - f_1| = f_0\frac{V_p}{c} = \frac{V_p}{\lambda_0}. \tag{5.38}$$

Thus if one were to measure the frequency $F_Д$, then one can determine the speed V_p of the travel of the receiver and transmitter relative to each other in a radial direction:

$$V_p = \frac{c}{f_0}F_Д = \lambda_0 F_Д. \tag{5.39}$$

Technically for an accurate measurement of the difference frequency $F_Д$, it should have a sufficiently large magnitude (hundreds and thousands of Hz).

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* If the measuring equipment of speed V_p and the measuring equipment of distance D are at the same point, then $V_p = \dot{D}$.

Therefore, the measurement of the speed of the rocket with the help of the Doppler effect is carried out on radio waves of the ultra-high frequency range. To measure the frequency $F_{Д}$ it is necessary that at the point of measurement the frequency of oscillations radiated by the transmitter be known with high accuracy. An analogous requireme-t, as is known (see § 5.2), appears in systems of measurement of distance, and its satisfaction in the measurement of speed can be carried out by the same methods as in range finders, by the installation on board the rocket and at the ground station of two high-stability generators or by means of re-emission of signals.

Let us examine how great the relative stability of generators should be if operation of the system is carried out by the first method. The instability of the frequency of generators installed at control center and on the rocket will lead to the appearance of a false difference of frequencies ΔF, which will cause the error ΔV_p in the measurement of speed. According to the formula (5.38) it is possible to write

$$\frac{F_Д + \Delta F}{f_0} = \frac{V_p + \Delta V_p}{c}. \tag{5.40}$$

From the relation (5.40) it follows that

$$\frac{\Delta F}{f_0} = \frac{\Delta V_p}{c}. \tag{5.41}$$

The latter expression permits determining the permissible magnitude relative detuning of the generators $\frac{\Delta F}{f_0}$. If one were to assume that the error of measurement of the speed caused by instability of generators should not exceed 0.1 m/s, then by the formula (5.41) we will obtain $\frac{\Delta F}{f_0} \doteq 3 \cdot 10^{-10}$. This relative frequency drift of ground and flight generators should not be exceeded during the whole time of operation of the measuring system, which is very difficult to fulfill. Of all the known devices only atomic and molecular generators have such high stability of frequency. The application of generators in control systems is considered very promising [19].

If the system of measurement of speed uses signals re-emitted from the rocket, the requirements for stability of frequency are reduced. In these systems the Doppler effect appears twice — during reception of the signal on the rocket and during its reception in the control center. The expression connecting the radial speed of the rocket V_p and difference frequency $F_Д$ in the system with re-emission will be written in the following way:

$$V_p = \frac{\lambda_0 F_Д}{2}. \tag{5.42}$$

For the separation of direct and re-emitted signals in systems with active response we operate on two frequencies. One frequency is of the transmitter of ground station (in the control center) and the other — the flight responder (radio relayer).

The accuracy of measurement of radial speed in Doppler systems depends on the accuracy of the measurement of difference frequency and in systems with relaying practically does not depend on the stability of high-frequency generators. With operation of the system at frequencies 10^9-10^{10} Hz and the speed of flight of the rocket 5-7 km/s, the difference frequency attains a magnitude of hundreds of kHz. In order that the error of measurement of speed does not exceed the desired value, the measurement of this frequency must be produced with an error measured by Hz units.

The difference frequency can be measured by determining the number of periods of oscillation of this frequency for a fixed interval of time. For the convenience of measurement sinusoidal oscillations exceed in pulses similar to that as was described in § 5.2 (Fig. 5.11). The number of pulses in limits of the fixed interval of time will be proportional to the mean value of frequency $F_Д$ during the counting time. The output signal of the pulse counter can be represented in binary code and introduced into the computer of the command system of radio control.

Measurement of Angular Velocity

In radio control command systems the measurement of radial speed of the rocket is supplemented by a measurement of the angular velocity of rotation of line of sight of the rocket from the control center. We measure the angular velocity of rotation of line of signt in a vertical plane $\dot{\xi}$ and angular velocity of rotation of the projection of line of signt on the horizontal plane $\dot{\nu}$.

Measurement of angular velocities $\dot{\nu}$ and $\dot{\xi}$ is produced by the indirect method — time derivative of the angles of ν and ξ are found. In practice we use finite increments and calculate, for example $\dot{\nu}$, by the formula

$$\dot{\nu} = \frac{\nu(t_2) - \nu(t_1)}{t_2 - t_1}, \tag{5.43}$$

where $\nu(t_1)$ and $\nu(t_2)$ are values of angle ν at instants t_1 and t_2. Angles ν and ξ for the calculation of angular velocities are determined with the help of the same systems as those which were described in § 5.3. However, for obtaining the necessary accuracy of measurement of angular velocities these systems should have different characteristics than during the measurement of angles.

With amplitude direction finding accurate measurement of the angular velocity of rotation of the line of signt is possible if the antennas have an extremely sharp antenna radiation patter. This requires the application of antennas with large dimensions. Even for waves of the centimeter range the antenna should have dimensions of several meters. In technical literature devoted to contemporary control systems of ballistic missiles there is not encountered indications of the application of amplitude direction finders for the determination of angular velocities.

Measurement of angular velocity of rotation of the line of sight ν (or ξ) can be carried out by the phase method. For simplification we will assume that the angle of elevation $\xi \approx 0$ and angle ν is small, so that it is possible to assume $\sin \nu \approx \nu$. Under these conditions instead of formula (5.22) we will write

$$\varphi_1 = \frac{2\pi d}{\lambda_0}\nu,$$

whence

$$\dot{\nu} = \frac{\lambda_0}{2\pi d}\dot{\varphi}_1. \tag{5.44}$$

The relation between errors of measurement $\Delta\nu$ and $\Delta\dot{\phi}_1$ has the form

$$d = (2 \cdots 5)\,10^{3}\lambda_0. \tag{5.45}$$

If one were to assume that the angular velocity $\dot{\nu}$ is measured with an error not exceeding the magnitude of the order of 10^{-6} rad/s, and radiotechnical devices determined the speed of the change in difference of phases ϕ_1 with an error at several degrees per second, then it is possible by formula (5.45) to obtain the necessary length of the base of the system d:

$$\tau = \frac{d}{c}\nu.$$

Thus the phase system operating on radio waves of the three-centimeter range, for achievement of the necessary accuracy of measurement of angular velocities, should have a base with the length of 60-150 m. This value is considerably larger than the length of the base with measurement of angular coordinates of the rocket (see § 5.3). However, the creation of the phase system with a base at 60-150 m does not now present principal technical difficulties.

The pulse method of measurement of angular velocities of the line of sight consists in the following. At $\xi \cong 0$, instead of formula (5.14), it is possible to write

$$\Delta\dot{\nu} = \frac{\lambda_0}{2\pi d}\Delta\dot{\varphi}_1.$$

It is assumed that the angle ν is small. On the basis of the last expression

$$\dot{\nu} = \frac{c}{d}\dot{\tau}, \tag{5.46}$$

whence

$$\Delta\dot{\nu} = \frac{c}{d}\Delta\dot{\tau}. \tag{5.47}$$

Let us assume that the measurement of $\Delta\dot{\tau}$ is fulfilled with an error of 0.1 μs/s. On the basis of this value and the requirement that the error of measurement ν does not exceed 2.10^{-6} rad/s, it is possible to calculate by formula (5.47) the length of the base of the pulse system. It will be equal to $d \cong 1500$ km. The system of measurement is obtained bulky.

Measurement of quantities $\dot{\nu}$ and $\dot{\xi}$ can be fulfilled also by the frequency method. If one were to consider $\xi \cong 0$, then formula (5.19) can be replaced by the following:

$$\nu = \frac{\lambda_0}{2d_0}\cdot\frac{F_p}{F_M}.$$

From this formula there follows

$$\dot{\nu} = \frac{\lambda_0}{2d_0}\cdot\frac{\dot{F}_p}{F_M}. \tag{5.48}$$

Hence we find

$$\Delta\dot{\nu} = \frac{\lambda_0}{2d_0}\cdot\frac{\Delta\dot{F}_p}{F_M}. \tag{5.49}$$

In order that it was possible confidently to reveal during the short interval of time (for providing the required high-speed operation of the control system) the change in difference frequency F_p, this frequency should be changed with sufficiently great speed. Considering that the frequency shift F_p is multiple to the frequency of modulation F_M (see § 5.2), we will assume that ΔF_p should be numerically equal approximately to F_M Hz/s. Taking into account this condition, from the relation (5.49) one can determine that the base of the measuring system with frequency modulation should be equal to 100-200 km.

From a comparison of the different methods of measurement of angular velocity of rotation of the line of sighting advantages of phase systems are seen. Such systems at superhigh frequencies permit obtaining great accuracy of measurement of angular velocity with not too great a length of the base of the antennas.

System of Measurement of Radial Velocity

Let us examine in the form of an example the functional diagram of the equipment for measurement of radial velocity represented in Fig. 5.22 [4]. The equipment is included in flying range control of ballistic missiles and include a ground station and flight responder. Operation of the system is based on the Doppler effect.

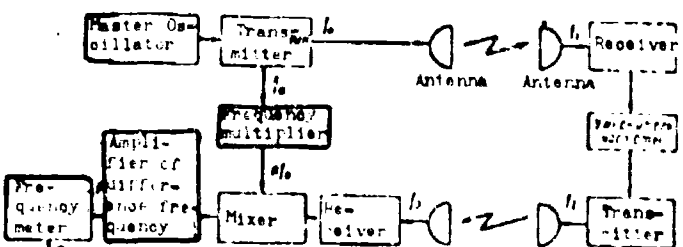

Fig. 5.22. Block diagram of the apparatus measuring the radial velocity of the rocket.

The high-stability master oscillator with a crystal stabilizer produces oscillations of the carrier frequency f_0. The transmitter of the ground station radiates these oscillations. Owing to the motion of the rocket the frequency of oscillations f_1 received on the rocket will be less than the frequency of the interrogation signal.

$$f_1 = f_0\left(1 - \frac{V_p}{c}\right), \tag{5.50}$$

where V_p is the radial velocity of the rocket, and c is propagation velocity of the radio waves.

The received signal is subjected to frequency multiplication. Frequency multiplication serves for the separation of interrogation and return signals, otherwise the signal coming from the rocket is difficult to separate against the background of the powerful radiation of the transmitter. The frequency of the signal can be multiplied by the number n, whole or fractional. After the multiplier we obtain oscillations with a frequency

$$f_2 = nf_1 = nf_0\left(1 - \frac{V_p}{c}\right), \tag{5.51}$$

which are transmitted to earth.

The ground receiver separates the signal by the frequency f_3, which differs due to the motion of the rocket from the value f_2:

$$f_3 = f_2\left(1 - \frac{V_p}{c}\right) = nf_0\left(1 - \frac{V_p}{c}\right)^2. \tag{5.52}$$

The ratio $\frac{V_p}{c}$ is many times less than unity. Therefore it is possible to consider

$$\left(1 - \frac{V_p}{c}\right)^2 \approx 1 - 2\frac{V_p}{c}. \tag{5.53}$$

Then the frequency of signal f_3 will be equal to

$$f_3 = nf_0\left(1 - 2\frac{V_p}{c}\right). \tag{5.54}$$

This signal is fed to the mixer together with the, multiplied frequency by n of the master oscillator f_0 oscillations. The difference frequency of the signal obtained at the output of the mixer will be equal

$$F_Д = nf_0 - nf_0\left(1 - 2\frac{V_p}{c}\right) = 2nf_0\frac{V_p}{c}. \tag{5.55}$$

The frequency Doppler meter $F_Д$ is fulfilled by the following functions. If the frequency $F_Д$ has a value corresponding to the calculated speed of the

rocket, the meter should issue a signal for the formation of a single command. There are formed two single commands, the preliminary and basic. Therefore, the meter should have tuning for two values of the difference frequency.

During the flight of the rocket the frequency $F_{Д}$, together with the speed of the rocket, continuously increases. The first signal of the frequency meter is formed when the speed of the rocket reaches the value at which there should be issued the preliminary command. The second signal of the meter corresponds to the basic command.

The station of the measurement of velocity, using the Doppler effect, determines the component of velocity of the rocket directed along the line of propagation of the radio wave. If it is necessary to measure the full speed of the rocket V, then ground the station is located in the guiding plane at such a point that on the segment of the measurement of velocity the trajectory of the rocket will coincide as fully as possible with the line AB (Fig. 5.23). In this case the radial velocity of the rocket V_p with sufficient accuracy be equal to the full speed V.

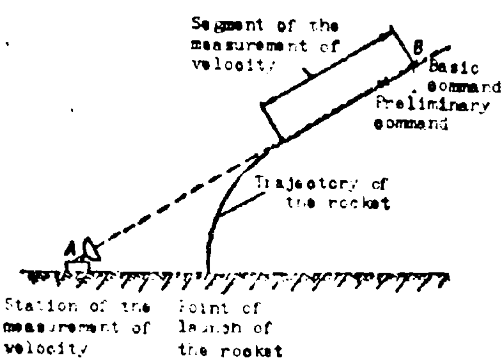

Fig. 5.23. Distribution of the station of measurement of velocity.